

# Anti-windup Coordination Strategy for Multivariable PID Control

Fernando Morilla  
UNED

Dpto. de Informática y Automática  
28040 Madrid (Spain)  
fmorilla@dia.uned.es

Juan Garrido  
Universidad de Córdoba  
Dpto. de Informática  
14071 Córdoba (Spain)  
p02gajuj@uco.es

Francisco Vázquez  
Universidad de Córdoba  
Dpto. de Informática  
14071 Córdoba (Spain)  
fvazquez@uco.es

## Abstract

*This paper presents a coordination strategy of anti-windup mechanisms based on conditioning by “input scaling”. It is specially designed to be used in distributed control systems with PID controllers. The study considers three cases of TITO control systems: decentralized control, centralized control with decoupler and centralized control with four PID. In them, the allowable region in the control signal space is transformed into an allowable region in the signal space of PID controllers. In this way, PID controllers are able to keep their integral term at a proper value when actuator saturates. The coordination strategy assures that the control signals are always inside these allowable control regions. The proposed methodology is applied to two representative processes.*

## 1. Introduction

The integral mode is included in the PID controllers to eliminate steady-state offset in presence of disturbances, under the supposition of unconstrained final elements (actuators). But sometimes the final elements reach their limits and the errors remain nonzero for long periods of time. The PID controllers must take it into account in order to avoid the reset (integral) windup problem and the corresponding very poor control performance.

There are many mechanisms to avoid this problem [1-2]. These mechanisms modify the standard PID algorithm without affecting its good performance during normal circumstances. They are generally designed to be applied in single-loop systems, but also they can be used in typical multiple-loop systems (cascade, selective, override, and ratio control) [3]. In more complex control systems, anti-reset windup should be included in every control algorithm that has integral mode, because most control strategies encounter limitations, perhaps infrequently, due to large changes in operating conditions [4].

The core idea used to protect systems against the negative effects of windup is turning the integrator off whenever the controller output reaches a limit. This idea

has been extended to MIMO systems by Goodwin et al. [2], assuming that the control law is biproper and of minimum phase. Subtle issues arise from the way the desired control is projected into the allowable region. They explore three possibilities (simple saturation, input scaling, and error scaling) using the point of view called conditioning. It is equivalent to answer the following question: What conditioned setpoint would have avoided producing a control signal beyond the allowable region? The figure 1 shows the conditioning anti-windup scheme for a biproper controller. The “Constraints” block must incorporate the model of the control signal constraints,  $K$ , is the high frequency gain matrix, and  $\bar{K}(s)$  is the transfer matrix that fulfils the following equation

$$K(s) = K_{\infty} + \bar{K}(s) \quad (1)$$

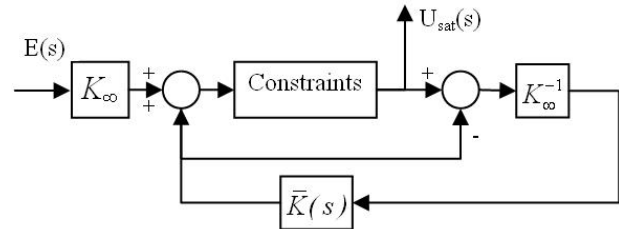


Figure 1. Conditioning anti-windup scheme.

We have tested this anti-windup scheme in simulation, controlling a boiler-turbine unit (3x3 nonlinear process) with a full matrix of nine PI controllers, obtaining good results in a wide operating range [5]. But this control strategy is not usually easy to implement using library components in distributed control systems.

In this paper coordination strategies of anti-windup mechanisms based on the conditioning form, specially designed to be used in distributed control systems with PID controllers, are presented. Our attention is limited to the three following typical TITO (two inputs - two outputs) control systems, but they can be easily extended to any number of inputs and outputs:

- The decentralized control, where two independent PID controllers are interacting through the control signals sent to the process [6].

- The centralized control with decoupler, where two independent PID controllers are interacting through the internal control signals sent to the decoupled process. In this option the two control signals are generated by the decoupler [7]. It is well known that there are difficulties to put in practice this configuration when constraints are present [3].

- The centralized control, where four PID controllers are collaborating by pair through the internal control signals, and where all PID controllers are interacting through the control signals sent to the process. In this last option two internal control signals will be delayed if the process model used to design the controllers has dead times [8].

In all these cases the constraints imposed on the control signal space are transmitted to controller signal space. Therefore the anti-windup mechanism in each PID controller will have to take into account the corresponding new range of operation, which can change dynamically. In Section 2, the allowable regions for TITO systems are analyzed in the different control signal spaces. Section 3 presents several coordination blocks in order to assure that the control signals are always inside the allowable control region. Two examples in simulation are illustrated in Section 4 showing the benefits of the coordination blocks. Finally, conclusions are presented in Section 5.

## 2. The allowable control region in TITO systems

In a SISO system the allowable control region is delimited by a minimum value and a maximum value. These values can be static, determined by the span of the control signal range, or dynamic if there are others constraints like slew rate on the control signal. The anti-windup mechanisms update these values using a model of the control signal constraints and notify to the controller if the calculated control signal is outside the allowable region and must be limited.

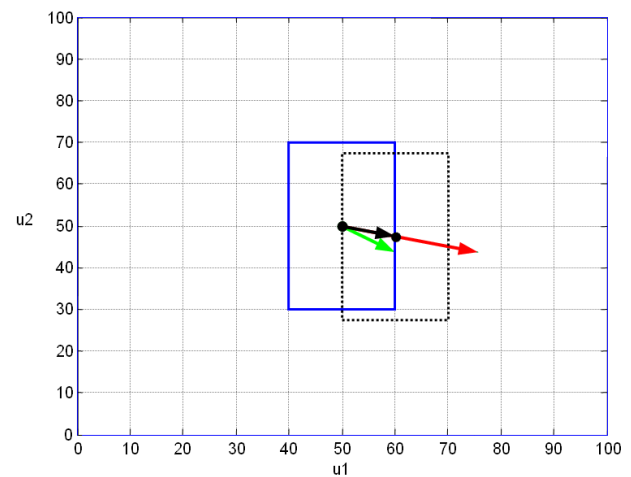
The anti-windup mechanisms for TITO system must consider that the limitation in a control signal will take effects in the other control signal because both are fed back through the process. The figure 2 shows this effect in a decentralized control system. In the control signal space ( $u_2-u_1$ ) the next elements have been represented:

- The operating point (50, 50) at the initial time.
- A large region, the static allowable region, delimited by the same range 0-100 in both signals.
- A more restrictive dynamic region delimited around the operating point by slew rate constraints of 10 units per sample time in the first signal and 20 units per sample time in the second one. This region is therefore the allowable control region at the initial time.
- A red arrow from the initial point to the point (75.60, 43.80) calculated by the controllers for the next

sample time. It can be observed that this point is outside the allowable region due to the first control signal is violating the constraints. This arrow is very important because it defines the direction in the control space selected by the controllers at the initial time.

- A green arrow from the initial point to the point (60.00, 43.80) obtained moving the first control signal to the border of the allowable region and maintaining the second control signal (“simple saturation”) [2].

- A black arrow from the initial point to the point (60.00, 47.58) obtained moving the first control signal to the border of the allowable region following the direction in the control space selected by the controllers. If this point is selected both control signals will be change (“input scaling”) [2], and the allowable control region for the next sample time is delimited by the dotted rectangle.

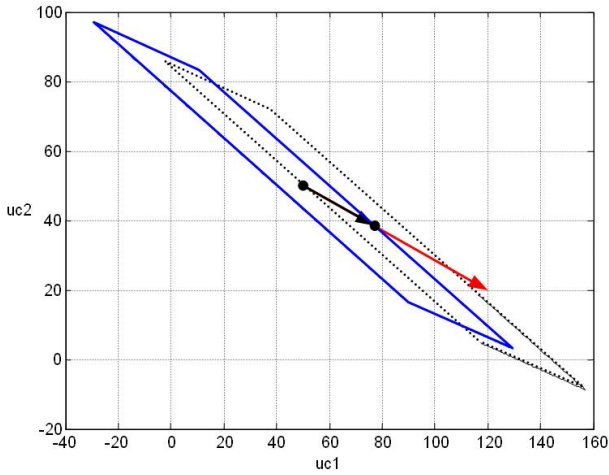


**Figure 2. Static and dynamic allowable control regions in a decentralized control system with saturation and slew rate constraints.**

The conditioning by “input scaling” is advised in multivariable system due to the directionality problem. If one or more control signals saturate independently, the direction of the new input vector does not coincide with the original input vector which counteracts the existing cross-couplings in the system. Therefore, it can give rise to strong and undesired coupling effects. With “input scaling” the input vector direction is preserved, so the directionality problem is reduced.

In a centralized control with decoupler the analysis is not so easier because there are two control spaces: the control signal space ( $u_2-u_1$ ) and the internal control signal space ( $u_{c2}-u_{c1}$ ). The control signals  $u_1$  and  $u_2$  are now the decoupler outputs, consequence of the signals  $u_{c2}$  and  $u_{c1}$  generated by the controllers. The figure 3 shows how the main elements of the figure 2 are transformed to the internal control space, assuming that the decoupler is a full open-loop stable matrix. Note that the dynamic rectangular regions of the control space are transformed

into quadrilateral dynamics regions in the internal control space. Also the static square region, not shown in the figure 3 is transformed into a quadrilateral region. Only the vectors related to “input scaling” are represented.



**Figure 3. Dynamic allowable control regions in a centralized control system with decoupler.**

The following equations summarize the transformations between points of figure 2 and 3:

$$\begin{aligned} u &= D(0)(u_c - u_i) + u_i \\ u_c &= D(0)^{-1}(u - u_i) + u_i \end{aligned} \quad (2)$$

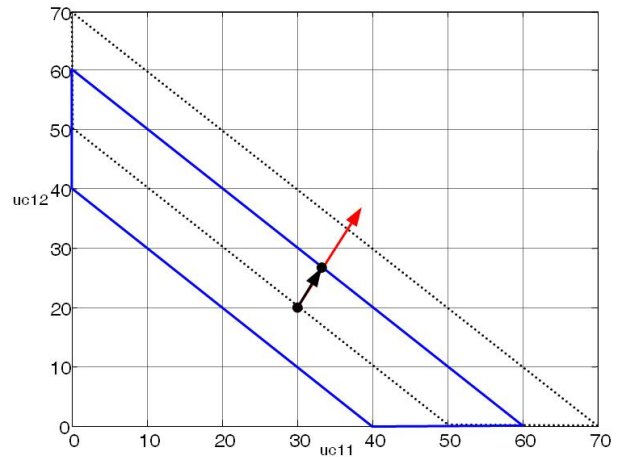
where  $u$  and  $u_c$  are arrays of two components with the respective static control values.  $D(0)$  is the static-gain of the linear decoupler  $D(s)$ , a  $2 \times 2$  matrix in this case. And  $u_i$  is the operating point at the initial time, the same in both spaces, but they could be different.

In a centralized control with four controllers there are three control spaces; the control signal space ( $u_2-u_1$ ), the internal control signal space ( $u_{c12}, u_{c11}$ ) and the internal control signal space ( $u_{c21}, u_{c22}$ ). The signals  $u_{c11}, u_{c12}, u_{c21}$  and  $u_{c22}$  are generated by the respective controllers, in such a way that

$$\begin{aligned} u_1 &= u_{c11} + u_{c12} \\ u_2 &= u_{c21} + u_{c22} \end{aligned} \quad (3)$$

Figure 4 shows how the main elements of the figure 2 are used in the internal control space for the first control signal. Note the dynamic range of the control signal  $u_1$  is

transformed now into a dynamic trapezoidal region. But also they could be triangular, it happens when the minimum value is zero, for example with the static square region, not shown in the figure. The initial point  $u_1=50$  is obtained with the couple  $(u_{c11}, u_{c12})=(30, 20)$ , the point  $u_1=75.60$  is calculated with the couple  $(38.60, 37)$  and the scaled point  $u_1=60$  limits the couple to  $(33.36, 26.64)$ .

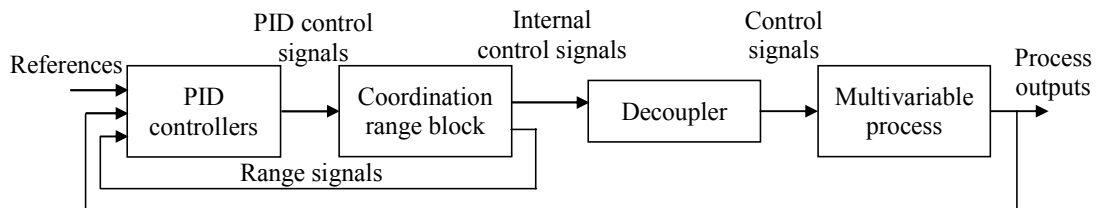


**Figure 4. Dynamic allowable control regions in a centralized control system with four PID controllers.**

### 3. Coordinating ranges

Assuming that all PID controllers collaborating in a multivariable control strategy have their own anti-windup mechanisms, in this paper it is proposed to use a coordination block between the controllers and the process or decoupled process. The coordination block will assure that the control signals send to the process, or to the decoupled process, are inside the allowable control region. But also it will protect the control system against the negative effects of windup generating the constraints for each controller. The figure 5 shows the more general block diagram corresponding to a centralized control with decoupler. But how does the coordination block make its task?

1°) The coordination block knows the constraints (minimum, maximum, and slew rate values) on the two control signals. Also it knows the last control signals



**Figure 5. General block diagram of a centralized control with decoupler**

send to the process (the operating point in figure 2) or the last internal control signals send to the decoupled process (the operating point in figure 3). Then it can determine the allowable region for the control signals (blue rectangle in figure 2) or the internal control signals (blue quadrilateral in figure 3).

2°) It requests the new control values to the controllers. Then, assuming that the control signals values will remain to these values, it can determine the future operating point. If a decoupler is used, this assumption is a bit restrictive because the dynamics of its elements are not taken into account. But it is not very restrictive either, because generally two elements of the decoupler are transfer functions and the other two are static gains.

3°) If the future operating point is outside the allowable region, the coordination block applies the “input scaling” in order to decide the new operating point. Otherwise, it accepts the future operating point. In both cases the coordination block can determine the new allowable region for the control signals (dotted rectangle in figure 2) or the internal control signals (dotted quadrilateral in figure 3). But if four PID controllers are controlling the process, the new allowable regions for the internal control signals (similar to the dotted trapeze in figure 4) must be also determined.

4°) The coordination block transmits the new operating point to the process, or decoupled process, and feeds back the new constraints to the controllers (range signals). In a rectangular allowable region, the constraints are clear. In quadrilateral and trapezoidal allowable regions it uses the constraints determined by the smallest rectangle centred in the operating point. Therefore each PID controller receives the most restrictive range for its output.

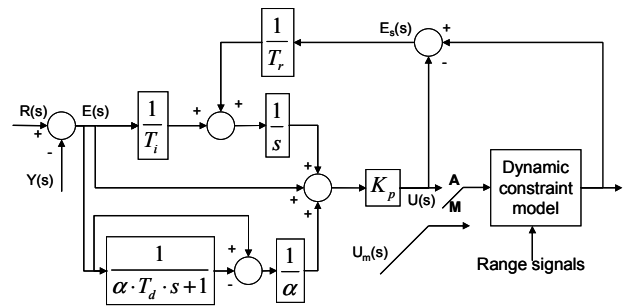
To test the previous anti-windup coordination strategy for TITO processes, the following S-functions have been developed in Simulink:

**1- PIDdiscrete awd:** S-function to simulate the digital PID algorithm with anti-windup based on dynamic constraints. The block interface consists of:

- Input signals: reference, process variable, manual control signal, operation mode, and control constraint signals (minimum, maximum, slew rates).
- Output signals: control signal.
- Parameters: sample time, proportional gain, integral time constant, derivative time constant, initial reference, initial control signal and derivative time noise filter constant.

This PID control is the digital implementation obtained from the structure of the continuous PID control depicted in figure 6. As it has been explained previously, the constraints model changes dynamically according to the new constraints which are sent by the coordination block in the range signals. The constant time  $T_r$  is used in the anti-windup mechanism to keep the

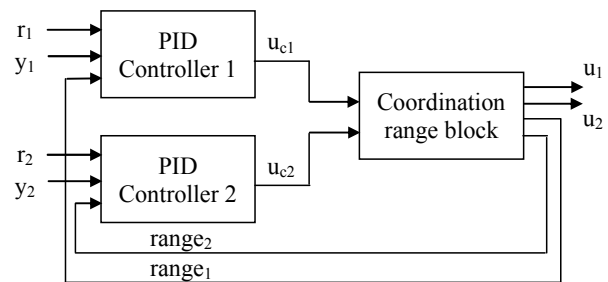
integral term at a proper value when the actuator saturates.



**Figure 6. PID control structure.**

**2- AWDecentralized:** S-function to coordinate two PID controllers collaborating in a decentralized control. It is placed between the controllers and the process (figure 7). Its interface consists of:

- Input signals: PID control signals.
- Output signals: control signals for the process and constraints for the two PID controllers.
- Parameters: sample time, initial control values, and constraints for the first control signal and for the second one.



**Figure 7. PID controllers collaborating in a decentralized control.**

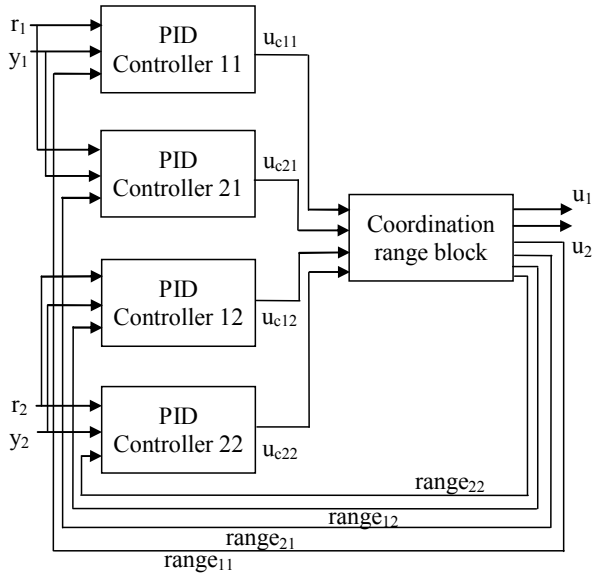
**3- AWDecoupler:** S-function to coordinate two PID controllers collaborating with a decoupler in a centralized control. It is placed between the two controllers and the decoupler. It works as the previous function but in the internal control signal space. Its interface consists of:

- Input signals: PID control signals.
- Output signals: internal control signals for the decoupler and constraints for the two PID controllers.
- Parameters: sample time, initial control values, static gain of the decoupler, constraints for the first control signal and the second one.

**4- AWCentralized:** S-function to coordinate four PID controllers collaborating in a centralized control. It is placed between the four PID controllers and the process (figure 8). Its interface consists of:

- Input signals: control signals of the four PID controllers.

- Output signals: control signals for the process and constraints for the four PID controllers.
- Parameters: sample time, initial control values, constraints for the first control signal and the second one.



**Figure 8. Four PID controllers collaborating in a centralized control.**

These coordination blocks can be easily implemented from library components in distributed control systems. Then, in order to get all their functionality, it is only necessary that the PID controllers have also an anti-windup mechanism based on dynamic constraints.

## 4. Examples

The proposed strategies are tested in simulation in two of the most cited processes.

### 4.1. Niederlinski's system

The process (4), originally proposed by Niederlinski [9], has become a classic in literature about decentralized and centralized control. It is a strong interacting process, with relative gain array (RGA) next to 0.5.

$$G(s) = \frac{I}{(0.1s + 1)(0.2s + 1)^2} \begin{pmatrix} 0.5 & -1 \\ 1 & 2.4 \\ & 0.5s + 1 \end{pmatrix} \quad (4)$$

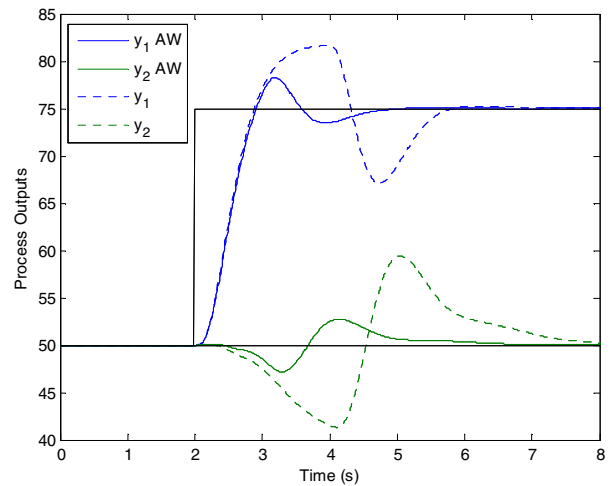
In this example the process is controlled by four PI controllers collaborating in a centralized control. The full PI controller (5) is designed to decouple the system and get a phase margin of  $60^\circ$  in both loops.

$$K_{PI} = \begin{pmatrix} 0.03 + \frac{2.60}{s} & 0.70 + \frac{0.97}{s} \\ -0.70 - \frac{0.97}{s} & 0.30 + \frac{0.49}{s} \end{pmatrix} \quad (5)$$

To test the proposed anti-windup coordination strategy, the system is subjected to the following constraints in both manipulated variables: saturation range between 20 and 80 units and slew rate limits of  $\pm 5$  units per sample time.

In the simulation, a step from 50 to 75 units in the first reference is carried out. The initial operation point is (50, 50) both outputs and inputs. The sample time is 0.01 seconds.

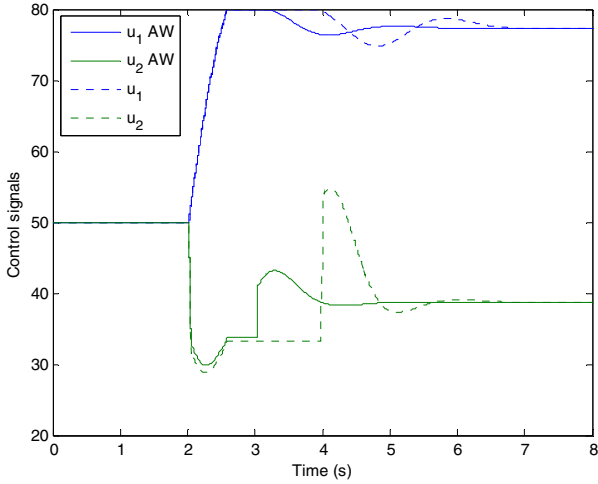
Figure 9 shows the output response of the closed-loop system when no anti-windup scheme is used, and when anti-windup mechanism with range coordination is applied. In this last case the response is better, with a faster reference tracking and less coupling effects.



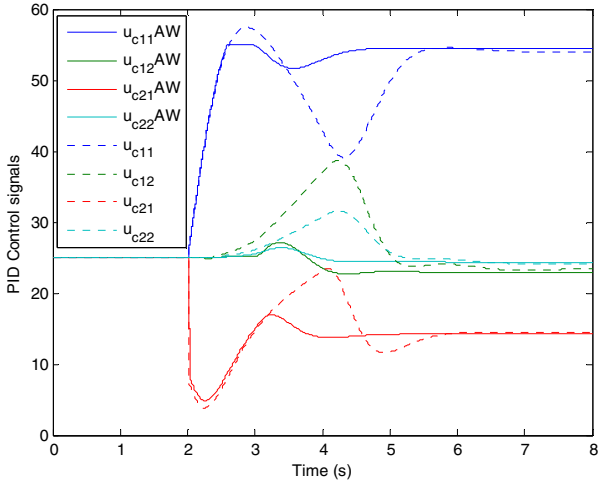
**Figure 9. Output responses of the Niederlinski's system with anti-windup ( $y_1$ AW,  $y_2$ AW) and without it ( $y_1$ ,  $y_2$ ).**

Figure 10 shows the comparison of the control signals in the two previous cases. They do not exceed the imposed constraints.

In the figure 11 the internal control signals of the four PI controllers in both cases are compared. It can be appreciated how the global constraints are transmitted to the signals  $u_{c11}$  and  $u_{c22}$  when the proposed methodology is used.



**Figure 10. Control signals of the Niederlinski's system with anti-windup ( $u_{1AW}$ ,  $u_{2AW}$ ) and without it ( $u_1$ ,  $u_2$ ).**



**Figure 11. PID control signals of the Niederlinski's system with anti-windup ( $u_{cijAW}$ ) and without it ( $u_{cij}$ ).**

#### 4.2. Wood and Berry column

The Wood-Berry binary distillation column plant [10] is a multivariable system that has been studied extensively. The process is a strong interacting process with RGA next to 2, and with important delays in its elements. It is described by the transfer matrix:

$$G(s) = \begin{pmatrix} \frac{12.8}{16.7s+1} e^{-s} & \frac{-18.9}{21.0s+1} e^{-3s} \\ \frac{6.6}{10.9s+1} e^{-7s} & \frac{-19.4}{14.4s+1} e^{-3s} \end{pmatrix} \quad (6)$$

In this example the controller consists of a decoupler  $D(s)$  (7) and a PI decentralized control  $K_d(s)$  (8), which make up a full centralized control.

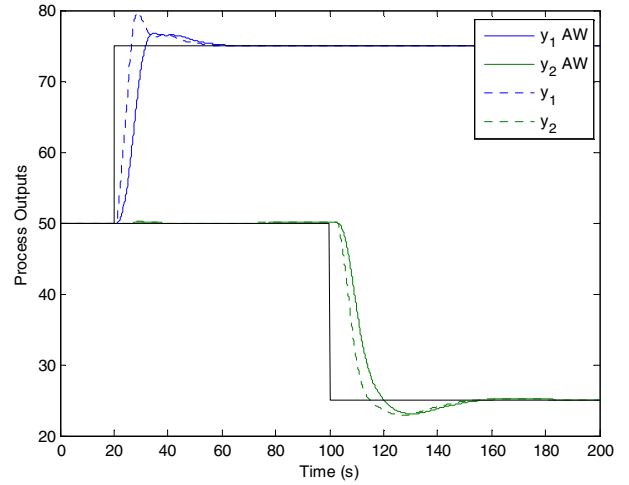
$$D(s) = \begin{pmatrix} 1 & \frac{1.48(16.7s+1)}{21s+1} e^{-2s} \\ \frac{0.34(14.4s+1)}{10.9s+1} e^{-4s} & 1 \end{pmatrix} \quad (7)$$

$$K_d(s) = \begin{pmatrix} 0.18 + \frac{0.468}{s} & 0 \\ 0 & -0.06 - \frac{0.2181}{s} \end{pmatrix} \quad (8)$$

The first PI controller is tuned to get a phase margin of  $60^\circ$  and a gain margin of 4 in the first loop. The second PI is tuned to get a phase margin of  $60^\circ$  and a gain margin equal to 3 in the other loop.

In the simulation, there are a step from 50 to 75 units in the first reference at  $t=20$  seconds and a step from 50 to 25 units in the second setpoint at  $t=100$  seconds. The initial operation point is (50, 50) both outputs and inputs. The sample time is 0.01 seconds.

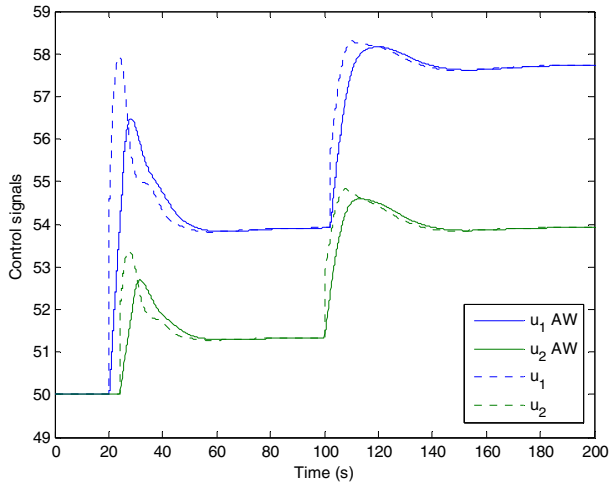
Figure 12 shows the output response of the closed-loop system without constraints and the output response when a slew rate constraint of  $\pm 0.1$  per sample time is imposed in both control signals. This limitation makes the response of the second case a bit slower, but both cases show a very good decoupling.



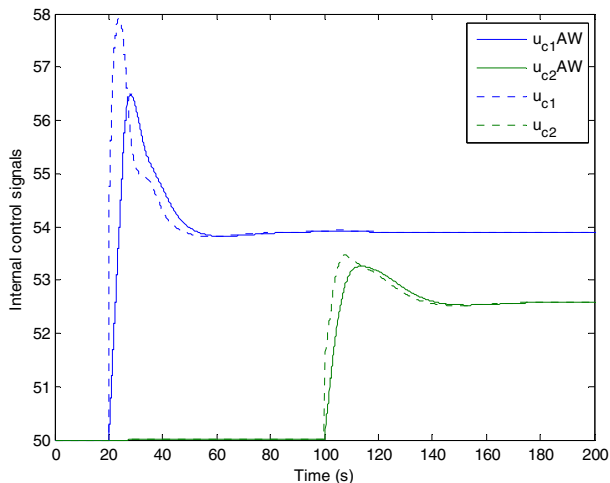
**Figure 12. Output responses of the Wood and Berry column without constraints ( $y_1$ ,  $y_2$ ) and with constraints plus anti-windup ( $y_{1AW}$ ,  $y_{2AW}$ ).**

In the figure 13 the control signals applied to the process are compared. It can be appreciated that the slew rate constraints are fulfilled.

Figure 14 shows the comparison of the internal control signals, which are generated by the two PI controllers in order to fulfill the slew rate constraints.



**Figure 13. Control signals of the Wood and Berry column without constraints ( $u_1$ ,  $u_2$ ) and with constraints plus anti-windup ( $u_{1AW}$ ,  $u_{2AW}$ ).**



**Figure 14. Internal control signals of the Wood and Berry column without constraints ( $u_{c1}$ ,  $u_{c2}$ ) and with constraints plus anti-windup ( $u_{c1AW}$ ,  $u_{c2AW}$ ).**

## 5. Conclusions

A coordination strategy of anti-windup mechanisms based on conditioning by “input scaling” has been proposed for TITO control systems. It has been tested in simulation with two representative processes using PID controllers and coordination blocks developed as S-functions in Simulink. The coordination strategy works properly in three types of control systems (decentralized control, centralized control with decoupler and centralized control with four PID), assuring that the control signals are always inside the allowable control

regions. And it is prepared to be used with real processes and to be implemented from library components in distributed control systems.

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## References

- [1] K. J. Åström, T. Hägglund, *Advanced PID control*, ISA-The Instrumentation, Systems, and Automation Society, Research Triangle Park, NC 27709, 2005.
- [2] G. C. Goodwin, S. F. Graebe, M. E. Salgado, *Control System Design*, Prentice Hall, 2001.
- [3] F. G. Shinskey, *Process control systems: Application, design, and tuning*, 4<sup>th</sup> Edition, McGraw-Hill, 1996.
- [4] T. E. Marlin, *Process Control: Designing processes and control systems for dynamic performance*, McGraw Hill, 1995.
- [5] J. Garrido, F. Morilla, F. Vázquez, “Centralized PID Control by Decoupling of a Boiler-Turbine Unit”, Accepted in the European Control Conference 2009 to be held in Budapest, Hungary during August 23-26, 2009.
- [6] F. Vázquez, F. Morilla, S. Dormido, “An iterative method for tuning decentralized PID controllers”, Proceeding of the 14th IFAC World Congress, pp. 491-496, 1999.
- [7] F. Vázquez, F. Morilla, “Tuning decentralized PID controllers for MIMO systems with decoupling”, Proceeding of the 15th IFAC World Congress, pp. 2172-2178, 2002.
- [8] F. Morilla, F. Vázquez, J. Garrido, “Centralized PID Control by Decoupling for TITO Processes”, Proceedings of 13th IEEE International Conference on Emerging Technologies and Factory Automation, pp. 1318-1325, 2008.
- [9] A. Niederlinski, “A heuristic Approach to the design of linear multivariable interacting control systems”, *Automatica*, Vol. 7, pp. 691-701, 1971.
- [10] R. K. Wood, M. W. Berry, “Terminal composition control of a binary distillation column”, *Chemical Engineering Science*, 28, pp. 1707-1717, 1973.