Centralized PID Control by Decoupling for TITO Processes

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Abstract

This paper presents a new methodology to design multivariable PID controllers for two-input and two-output systems. The proposed control strategy, which is centralized, combines four PID controllers plus two possible delays and two P controllers. The proportional gains in the P controllers act as tuning parameters in order to modify the behavior of the loops almost independently. The design procedure consists of three steps: first, an ideal decoupler including integral action is determined. Second, the decoupler is approximated with four PID controllers plus two possible delays. Third, the proportional gains are tuned to achieve the specified performance. The proposed method is applied to three representative processes.

1. Introduction

Generally, most industrial processes are multivariable systems. Two-input two-output (TITO) system is one of the most prevalent categories of multivariable systems, because there are real processes of this nature or because a complex process has been decomposed in 2×2 blocks [1, 2, 3, 4] with non negligible interactions between its inputs and outputs.

When the interactions in different channels of the process are modest, a diagonal controller (decentralized control) is often adequate. Nevertheless, when interactions are significant, a full matrix controller (centralized control) is advisable. The Figs. 1 and 2 show two different approaches of centralized control for TITO processes. The first of them uses a decoupling network with four $d_{ij}(s)$ and a decentralized controller with two $k_i(s)$. The second, in Fig. 2, uses a pure centralized strategy, with four $k_{ij}(s)$.

The block diagram in Fig. 1 has received considerable attentions in both control theory and industrial practice for several decades using a decentralized PI or PID [5]. However, it is more difficult to find works using PID controllers with the diagram in Fig. 2. There are no available commercial products using this kind of controllers. In distributed control systems it is possible to build it from library components, but it is necessary to coordinate manual to automatic mode changes (or automatic to manual) in every pair of PID blocks. Lieslehto, in [6], presented an n×n centralized control and its particularization to PID controllers based on Internal Model Control (IMC) SISO design. A more experimental work [7] approaches the multivariable PID controller tuning as an optimization problem, where it is necessary to define the desired closed loop transfer function matrix. Both works present several limitations when the process incorporates significant time delays.

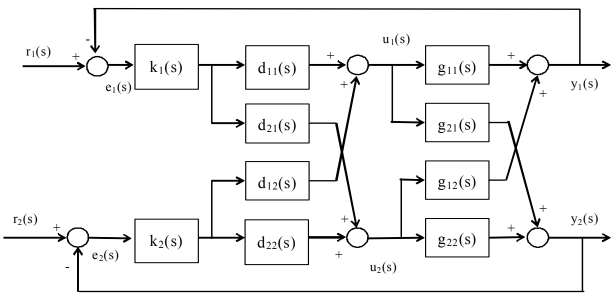

Figure 1: 2×2 Centralized control combining a decoupler and a decentralized controller.

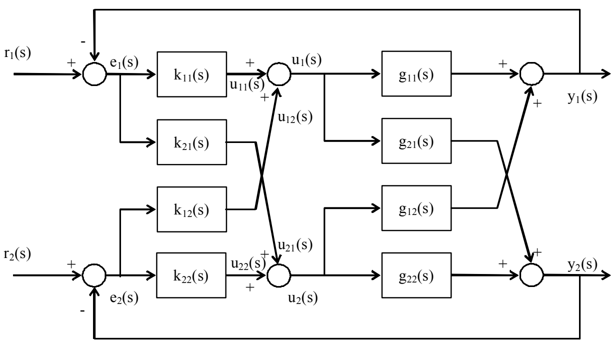

Figure 2: 2×2 centralized control with four controllers.

There is another important reason to research ways of dealing with interaction at the loop level [1]. Model predictive control (MPC) is becoming the standard technique to solve multivariable control problems in the process industry. However practically all MPC systems are operating in a supervisory mode with sampling times that are longer than in the PID controllers at the lower level. And there are some difficulties in dealing with the interaction at the MPC level because the bandwidths of the MPC loops are limited. Therefore, the centralized control using PID controllers, at least in the 2×2 case, is an interesting strategy in the process industry.

This paper presents a new approach to decoupling problem for 2×2 stable or integrator plants with time delays. First, an ideal decoupler including integral action is determined. Second, the decoupler is approximated with four PID controllers plus two possible delays. Third, the proportional gains are tuned to achieve the specified performance. The PID decoupling control is discussed in Section 2. The proposed method is applied to three representative processes in Section 3, followed by conclusions in Section 4.

2. PID decoupling control

Consider the conventional unity output feedback 2×2 control system in Fig. 2, where the transfer matrix of the plant G(s) could have a common dynamic $g_c(s)$ in all its elements $g_{ij}(s)$

$$G(s)=\begin{pmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s)\end{pmatrix} = g_c(s)\begin{pmatrix} \hat{g}_{11}(s) & \hat{g}_{12}(s) \\ \hat{g}_{21}(s) & \hat{g}_{22}(s)\end{pmatrix} = g_c(s)\,\widehat{G}(s) \quad (1)$$

and the controller K(s) is a 2x2 full-cross coupled multivariable transfer matrix

$$\begin{pmatrix} u_1(s) \\ u_2(s)\end{pmatrix} = \begin{pmatrix} k_{11}(s) & k_{12}(s) \\ k_{21}(s) & k_{22}(s)\end{pmatrix}\begin{pmatrix} r_1(s) - y_1(s) \\ r_2(s) - y_2(s)\end{pmatrix} \quad (2)$$

There are multiple alternatives to design the four elements $k_{ij}(s)$. Most of them [3, 4, 8, 9, 10] are based in scheme of Fig. 1. In these cases, as it is proposed in [11], first the decoupler D(s) and then the two $k_i(s)$ elements of the diagonal controller $K_d(s)$ are designed. The resulting centralized controller is $K(s)=D(s)K_d(s)$.

Other methodologies [5, 6, 12] under the paradigm of "decoupling control" propose to find a K(s) such that the closed loop transfer matrix $G(s)K(s)[I + G(s)\,K(s)]^{-1}$ is decoupled over some desired bandwith. This goal is ensured if the open loop transfer matrix L(s)=G(s)K(s) is diagonal. For this reason, the techniques used in decoupling control are very similar to the techniques used to design decouplers.

Then, assuming that the open loop transfer matrix should be diagonal $L(s) = \mathrm{diag}(l_1(s),l_2(s))$, the following expression is obtained for the controller by decoupling,

$$K = G^{-1}\,L = \frac{1}{g_c\left|\widehat{G}\right|}\begin{pmatrix} \hat{g}_{22} & -\hat{g}_{12} \\ -\hat{g}_{21} & \hat{g}_{11}\end{pmatrix}\begin{pmatrix} l_1 & 0 \\ 0 & l_2\end{pmatrix}$$

$$K = \frac{1}{g_c\left|\widehat{G}\right|}\begin{pmatrix} \hat{g}_{22}\,l_1 & -\hat{g}_{12}\,l_2 \\ -\hat{g}_{21}\,l_1 & \hat{g}_{11}\,l_2\end{pmatrix} = \begin{pmatrix} \tilde{g}_{11}^{-1}\,l_1 & \tilde{g}_{21}^{-1}\,l_2 \\ \tilde{g}_{12}^{-1}\,l_1 & \tilde{g}_{22}^{-1}\,l_2\end{pmatrix} \quad (3)$$

where the complex variable s has been omitted, where the 2×2 determinant $\left|\widehat{G}\right|$ is

$$\left|\widehat{G}\right| = \hat{g}_{11}\,\hat{g}_{22} - \hat{g}_{12}\,\hat{g}_{21}$$

and where the four transfer functions

$$\tilde{g}_{11} = \frac{g_c\left|\widehat{G}\right|}{\hat{g}_{22}} \quad \tilde{g}_{12} = \frac{g_c\left|\widehat{G}\right|}{-\hat{g}_{21}} \quad \tilde{g}_{21} = \frac{g_c\left|\widehat{G}\right|}{-\hat{g}_{12}} \quad \tilde{g}_{22} = \frac{g_c\left|\widehat{G}\right|}{\hat{g}_{11}}$$

are the equivalent processes for the four decoupled SISO loops [5] controlled respectively by k_{11}, k_{21}, k_{12} and k_{22}.

From equations (2) and (3), the following compact decoupling equation is obtained

$$k_{ji} = \tilde{g}_{ij}^{-1}\,l_i \quad ; \quad i=1,2 \text{ and } j=1,2 \quad (4)$$

Note that this equation has a great similarity with Affine Parameterization [12] and IMC [13]. In order to obtain the four k_{ij}, it is only necessary to specify the two transfer functions $l_i(s)$. Next subsections treat different design considerations to solve the presented problem.

2.1. More about the $\tilde{g}_{ij}$

It is necessary to obtain a new approach to decoupling problem for 2×2 stable plants with time delays. It is assumed that the common dynamic and the four elements are strictly proper, stable rational transfer functions plus possible time delays

$$g_c = \frac{b_c}{a_c}\,e^{-s\tau_c} \quad ; \quad \hat{g}_{ij} = \frac{\hat{b}_{ij}}{\hat{a}_{ij}}\,e^{-s\hat{\tau}_{ij}} \quad (5)$$

Then, the four transfer functions $\tilde{g}_{ij}$ are given by

$$\tilde{g}_{11} = \frac{n}{a_c\,\hat{a}_{11}\,\hat{a}_{12}\,\hat{a}_{21}\,\hat{b}_{22}\,e^{-s\hat{\tau}_{22}}} \quad (6)$$

$$\tilde{g}_{12} = \frac{-n}{a_c\,\hat{a}_{11}\,\hat{a}_{12}\,\hat{b}_{21}\,e^{-s\hat{\tau}_{21}}\,\hat{a}_{22}} \quad (7)$$

$$\tilde{g}_{21} = \frac{-n}{a_c\,\hat{a}_{11}\,\hat{b}_{12}\,e^{-s\hat{\tau}_{12}}\,\hat{a}_{21}\,\hat{a}_{22}} \quad (8)$$

$$\tilde{g}_{22} = \frac{n}{a_c\,\hat{b}_{11}\,e^{-s\hat{\tau}_{11}}\,\hat{a}_{12}\,\hat{a}_{21}\,\hat{a}_{22}} \quad (9)$$

where

$$n = b_c e^{-s\tau_c}\left(\hat{b}_{11}e^{-s\hat{\tau}_{11}}\,\hat{a}_{12}\,\hat{a}_{21}\,\hat{b}_{22}e^{-s\hat{\tau}_{22}} - \hat{a}_{11}\,\hat{b}_{12}e^{-s\hat{\tau}_{12}}\,\hat{b}_{21}e^{-s\hat{\tau}_{21}}\,\hat{a}_{22}\right) \quad (10)$$

After the corresponding zero-pole cancellations, the following general equation can be used

$$\tilde{g}_{ij} = \frac{\tilde{b}_{ij}}{\tilde{a}_{ij}}\,e^{-s\tilde{\tau}_{ij}} \quad (11)$$

where $\tilde{a}_{ij}$ and $\tilde{b}_{ij}$ have not common factors. The time delays $\tilde{\tau}_{ij}$ can be determined as following equations show:

$$\tau(n) = \tau_c + \mathit{min}\,(\hat{\tau}_{11}+\hat{\tau}_{22}, \hat{\tau}_{12}+\hat{\tau}_{21}) \quad (12)$$

$$\tilde{\tau}_{ii} = \tau(n) - \hat{\tau}_{jj} \quad i=1,2\ ;\ j\#\,i \quad (13)$$

$$\tilde{\tau}_{ij} = \tau(n) - \hat{\tau}_{ji} \quad i=1,2\ ;\ j\#\,i \quad (14)$$

where $\tau(f(s))$ represents the time delay of a generic function f(s) and *min* represents the minimum function.

2.2. How to specify the $l_i(s)$

Every open-loop transfer function $l_i(s)$ used in equation (4) must take into account the dynamic of two processes $\tilde{g}_{ii}$ and $\tilde{g}_{ij}$, the achievable performance specifications of the corresponding SISO closed-loop system, and, not less important, that both controllers k_{ii} and k_{ji} must be realizable. Due to the time delay of the processes, the largest one must be included in the open-loop transfer function $l_i(s)$. Since the closed-loop must be stable and without steady-state errors due to setpoint or load changes, the open-loop transfer function $l_i(s)$ must contain an integrator. Then, the following general expression for $l_i(s)$ is proposed:

$$l_i(s) = k_i\ \bar{l}_i(s)\ \frac{1}{s}\ e^{-s\tau_i} \qquad \text{with } \tau_i \geq 0 \quad (15)$$

where

$$\tau_i = max\ (\tilde{\tau}_{ii}, \tilde{\tau}_{ij}) = \tau\ (n) - min\ (\hat{\tau}_{jj}, \hat{\tau}_{ji})\ \ i=1,2;\ \ j\#i \quad (16)$$

Parameter k_i becomes a tuning parameter in order to met design specifications and the $\bar{l}_i(s)$ must be a rational transfer function taking into account the not cancellable dynamic of $\tilde{g}_{ii}$ and $\tilde{g}_{ij}$. Note that this not cancellable dynamic is generally the same in the four equivalent processes $\tilde{g}_{ij}$. Next, the attention is directed to the three most common cases included in table 1.

Table 1. The three most common open-loop transfer functions according to the process to be controlled.

Process	$\bar{l}_i(s)$	$l_i(s)$
Stable and minimum phase	1	$\frac{k_i}{s}\ e^{-s\tau_i}$
Stable and non-minimum phase	$\frac{-(s-z)}{s+z}$	$\frac{-k_i\ (s-z)}{s\ (s+z)}\ e^{-s\tau_i}$
Stable and minimum phase plus integrator	$\frac{s+z_i}{s}$	$\frac{k_i\ (s+z_i)}{s^2}\ e^{-s\tau_i}$

2.3. How to determine the parameters of $l_i(s)$

This question is analyzed for each of the cases of table 1. The $l_i(s)$ parameters are determined depending on desired specifications of the closed-loop system.

Case 1: The equivalent processes are stable and minimum phase systems, all their dynamics are cancellable, so $\bar{l}_i(s)$ and $l_i(s)$ are given by the first row in table 1. It is enough to impose relative stability specification (gain or phase margins) to the function $l_i(s)$ in order to guarantee the stability of the closed-loop transfer function $h_i(s)=l_i(s)/(1+l_i(s))$. It can be shown that $l_i(s)$ gets the following stability margins at following frequencies:

$$\phi_m = 90 - \frac{180\ k_i\ \tau_i}{\pi} \quad ; \quad \omega_{cp} = k_i \quad (17)$$

$$A_m = \frac{\pi}{2\ k_i\ \tau_i} \quad ; \quad \omega_{cg} = \frac{\pi}{2\ \tau_i} \quad (18)$$

Then both stability margins are related by:

$$\phi_m = 90 - \frac{90}{A_m} \quad (19)$$

If phase margin ϕ_m (less than 90º) is specified or gain margin A_m (greater than 1), the value of k_i is directly determined as follows:

$$k_i = \frac{\pi(90 - \phi_m)}{180\ \tau_i} \quad (20)$$

$$k_i = \frac{\pi}{2\ A_m\ \tau_i} \quad (21)$$

Increasing k_i makes the loop faster but with smaller values of phase margin and gain margin. If no delay is present, $\tau_i = 0$, the function $l_i(s)=k_i/s$ shows a phase margin of 90º and an infinite gain margin, independent of k_i value. Thus, it is impossible to use specifications of relative stability to determine the value of k_i. In this case, the closed-loop transfer function has the typical shape of a first order system:

$$h_i(s) = \frac{\frac{k_i}{s}}{1 + \frac{k_i}{s}} = \frac{1}{T_i\ s + 1} \quad (22)$$

with time constant $T_i = 1/k_i$. Then, in order to determine k_i it is enough to specify the time constant of the closed-loop system. This situation coincides with the most common case in the synthesis by using the Affine Parametrization [12] and in the IMC methodology [13].

Case 2: The equivalent processes are stable and non-minimum phase systems. The common non-minimum phase zero cannot be cancelled, it must appears in $\bar{l}_i(s)$, so $\bar{l}_i(s)$ and $l_i(s)$ are given by the second row in table 1. In these conditions, it can be shown that $l_i(s)$ function gets the following gain margin

$$A_m = \frac{\omega_{cg}}{k_i} \quad (23)$$

at frequency ω_{cg}, that verifies the following equation

$$\tan\ (\omega_{cg}\tau_i) = \frac{1 - \frac{\omega_{cg}^2}{z^2}}{2\frac{\omega_{cg}}{z}} \quad (24)$$

The proposal of this paper is to determine ω_{cg} from equation (24) and to obtain k_i from equation (23) in order to achieve the desired value of A_m.

If no delays are present, $\tau_i = 0$, the A_m specification can be replaced by time response specifications, because closed loop transfer function is

$$h_i(s) = \frac{\frac{-k_i\ (s-z)}{s\ (s+z)}}{1 - \frac{k_i\ (s-z)}{s\ (s+z)}} = \frac{-k_i\ (s-z)}{s^2 + (z-k_i)\ s + k_i\ z} \quad (25)$$

Its poles are characterized by the undamped natural frequency and the damping factor

$$\omega_n = \sqrt{k_i\ z} \qquad ; \qquad \delta = \frac{z - k_i}{2\sqrt{k_i\ z}} \tag{26}$$

and then, it is possible to fix the value δ with $k_i<z$.

Case 3: The equivalent processes are stable, except in s=0, and minimum phase systems. The pole in s=0 must appear in $\bar{l}_i(s)$; then $\bar{l}_i(s)$ and $l_i(s)$ are given by the third row in table 1. In these conditions, it can be shown that $l_i(s)$ function gets the following gain margin

$$A_m = \frac{\omega_{cg}^2}{k_i\sqrt{\omega_{cg}^2 + z_i^2}} \tag{27}$$

at frequency ω_{cg}, that verifies the following equation

$$\text{arc tg}\ \frac{\omega_{cg}}{z_i} - \omega_{cg}\ \tau_i = 0 \tag{28}$$

Author proposes to fix the value of the zero z_i, and subsequently to determine the gain margin frequency ω_{cg} from equation (28) and k_i from equation (27).

If there is not delay, $\tau_i = 0$, the A_m specification can be replaced by time response specifications, because closed-loop transfer function becomes

$$h_i(s) = \frac{\dfrac{k_i\ (s+z_i)}{s^2}}{1 + \dfrac{k_i\ (s+z_i)}{s^2}} = \frac{k_i\ (s+z_i)}{s^2 + k_i\ s + k_i\ z_i} \tag{29}$$

Its poles are characterized by the undamped natural frequency and the damping factor

$$\omega_n = \sqrt{k_i\ z_i} \qquad ; \qquad \delta = \sqrt{\frac{k_i}{4\ z_i}} \tag{30}$$

Subsequently, if the value of z_i is fixed, with k_i it is possible to modify the values of ω_n and δ. Particularly, it is sufficient to select $k_i=4z_i$ in order to achieve poles with critical damping (δ=1) and $\omega_n=2z_i$.

2.4. About the controllers $k_{ij}(s)$

Substituting equations (11) and (15) into (4) yields

$$k_{ji}(s) = k_i\ \frac{\tilde{a}_{ij}(s)}{\tilde{b}_{ij}(s)}\ \bar{l}_i(s)\ \frac{1}{s}\ e^{-s(\tau_i - \tilde{\tau}_{ij})} \tag{31}$$

Then, the four resulting controller elements $k_{ji}(s)$ have integral action and, at least, two of them must delay their control actions as equations (13), (14) and (16) show:

$$i=1,2\ ;\ j \# i$$

$$\tau(k_{ii}) = \tau_i - \tilde{\tau}_{ii} = \hat{\tau}_{jj} - \mathit{min}\,(\hat{\tau}_{jj}, \hat{\tau}_{ji}) \begin{cases} 0 & \text{if } \hat{\tau}_{jj} \le \hat{\tau}_{ji} \\ \hat{\tau}_{jj} - \hat{\tau}_{ji} & \text{if } \hat{\tau}_{jj} > \hat{\tau}_{ji} \end{cases}$$

$$\tau(k_{ji}) = \tau_i - \tilde{\tau}_{ij} = \hat{\tau}_{ji} - \mathit{min}\,(\hat{\tau}_{jj}, \hat{\tau}_{ji}) \begin{cases} 0 & \text{if } \hat{\tau}_{jj} > \hat{\tau}_{ji} \\ \hat{\tau}_{ji} - \hat{\tau}_{jj} & \text{if } \hat{\tau}_{jj} \le \hat{\tau}_{ji} \end{cases} \tag{32}$$

Moreover the four elements $k_{ji}(s)$ are usually highly complicated and difficult to implement. Some authors [5, 8, 9, 10] suggest the use of model reduction techniques. In next subsection, these techniques are used in order to get PID structure.

2.5. Using PID structure

If it is intended that controllers become ideal PID plus delay, it is necessary to force the following structure in all controller elements

$$k_{ji}^{PID}(s) = \frac{k_i}{s}\left(K_{Dji}\ s^2 + K_{Pji}\ s + K_{Iji}\right) e^{-s\tau(k_{ji})} \tag{33}$$

where it appears the controller delay $\tau(k_{ji})$ and its proportional (K_{Pji}), integral (K_{Iji}) and derivative (K_{Dji}) gains. Also note that the same tuning parameter k_i appears in k_{1i} and k_{2i}.

Instead of reducing the controller given by (31) or (33), authors propose to remove the delay, the integrator and the respective k_i from the controller and to apply the model reduction to the inverse simplified expression m_{ji} as follows:

$$k_{ji}(s) = k_i\ \left(m_{ji}(s)\right)^{-1}\ \frac{1}{s}\ e^{-s\tau(k_{ji})}$$

$$m_{ji}(s) = \frac{\tilde{b}_{ij}(s)}{\tilde{a}_{ij}(s)}\ \left(\bar{l}_i(s)\right)^{-1} \cong \frac{b_o}{a_2\ s^2 + a_1\ s + a_o} \tag{34}$$

In this way, the controller gains will be $K_{Pji} = a_1/b_o$, $K_{Iji} = a_o/b_o$ y $K_{Dji} = a_2/b_o$.

2.6. Testing the design

In order to put in practice the design, it is recommended to use the centralized control structure of Fig. 1, where decoupling network includes the PID blocks and its corresponding delays, and where $k_1(s)$ and $k_2(s)$ are proportional controllers with gains k_1 and k_2 determined through the design. Then

$$K^{PID}(s) = D^{PID}(s)\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \tag{35}$$

This structure allows the design to be easily tested with TITO tool obtained in author web page (www.dia.uned.es/~fmorilla/fmg_english.htm) that, from its later version, is able to treat decouplers with delays.

3. Examples

The proposed methodology is efficient with two of the most cited processes [14, 15] but also with the following examples that consider three important cases: big delay, non-minimum phase process and integrator process.

3.1. Wood and Berry column

The Wood-Berry binary distillation column plant [16] is a multivariable system that has been studied extensively. The process has important delays in its elements. It is described by the transfer matrix:

$$G(s)=e^{-s}\begin{pmatrix} \dfrac{12.8}{16.7s+1} & \dfrac{-18.9}{21.0s+1}e^{-2s} \\ \dfrac{6.6}{10.9s+1}e^{-6s} & \dfrac{-19.4}{14.4s+1}e^{-2s} \end{pmatrix} \quad (36)$$

1) The four equivalent processes $\tilde{g}_{ij}$ resulting from (6)-(10) are stable and minimum phase systems. Here, in order to save space, only $\tilde{g}_{11}$ is shown

$$\frac{-12.8\ 19.4(21.0s+1)(10.9\ s\ +\ 1)\ +18.9\ \ 6.6(16.7s+1)(14.4s+1)\ e^{-6s}}{-19.4(21.0s+1)(10.9s+1)(16.7s+1)}e^{-s} \quad (37)$$

The two open-loop transfer functions $l_1(s)$ y $l_2(s)$ are chosen following the first row of table 1. Their respective time delays are determined from equations (12) and (16):

$$\tau(n) = 1 + min\,(0 + 2, 2 + 6) = 3$$
$$\tau_1 = 3 - min\,(2,6) = 3 - 2 = 1$$
$$\tau_2 = 3 - min\,(0,2) = 3 - 0 = 3$$

2) The delays for the controller elements follow from equation (32)

$$\tau(k_{11}) = 2 - min(6,2) = 0$$
$$\tau(k_{21}) = 6 - min\,(2,6) = 4$$
$$\tau(k_{12}) = 2 - min\,(0,2) = 2$$
$$\tau(k_{22}) = 0 - min\,(0,2) = 0$$

3) The exact calculus of $k_{11}(s)$ from equation (29) produces

$$\frac{-19.4(21.0s+1)(10.9s+1)(16.7s+1)\frac{k_1}{s}}{-12.8\ \ 19.4(21.0s+1)(10.9s+1)+18.9\ \ 6.6(16.7s+1)(14.4s+1)e^{-6s}} \quad (38)$$

Confirming that the ideal element $k_{11}(s)$ must not include time delay and it is difficult to implement. Then, approximating the inverse simplified expression m_{11}, see Fig. 3, as follows

$$\frac{-12.8\ 19.4(21.0s+1)(10.9s+1)+18.9\ 6.6(16.7s+1)(14.4\ s+1)e^{-6s}}{-19.4(21.0s+1)(10.9s+1)(16.7s+1)} \approx \frac{0.8061}{s+0.129} \quad (39)$$

the resultant PI controller is

$$k_{11}(s) \approx k_{11}^{PI}(s) = k_1\ \frac{s+0.129}{0.8061\ s} \quad (40)$$

After a PI approximation in all its elements, the integral decoupling matrix in following equation is conformed.

$$D_{WB1}^{PI}(s) = \begin{pmatrix} \dfrac{s+0.13}{0.81s} & -\dfrac{s+0.23}{1.50s}e^{-2s} \\ \dfrac{s+0.085}{1.57s}e^{-4s} & -\dfrac{s+0.165}{1.57s} \end{pmatrix} \quad (41)$$

A PID decoupler has also been tested, but derivatives gains were very low, and it became a PI.

The decoupler in equation (41) could be used in Fig. 1 to make several experiments on the Wood and Berry column, taking different values of k_1 and k_2. Fig. 4 shows the step response of the closed-loop system with $k_1 \cong 0.31$ and $k_2 \cong 0.17$ corresponding to margins $A_{m1}=5$ and $A_{m2}=3$ in (21). Both loops show an oscillatory behaviour, that can be reduced decreasing the gains k_1 and k_2, although the response becomes slower. Also note that there is some loop interaction, because of the controller approximation, but it is not relevant. This response is similar to the obtained by other authors [8, 9] using an ideal decoupler plus a decentralized PI controller. In the same figure, results from [9] are shown.

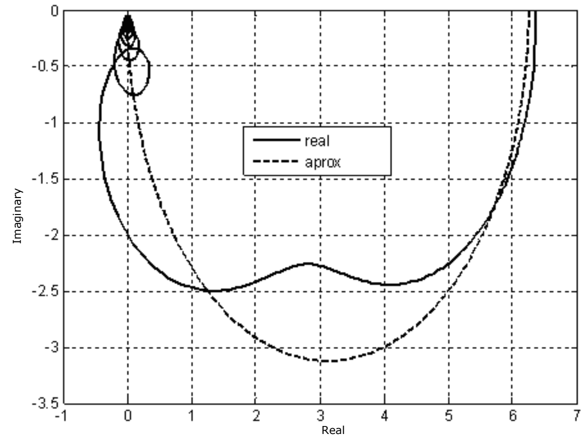

Figure 3: Frequency response (solid line) of $m_{11}(s)$ and approximated response (dashed line) of first order using (34) with a_2=0.

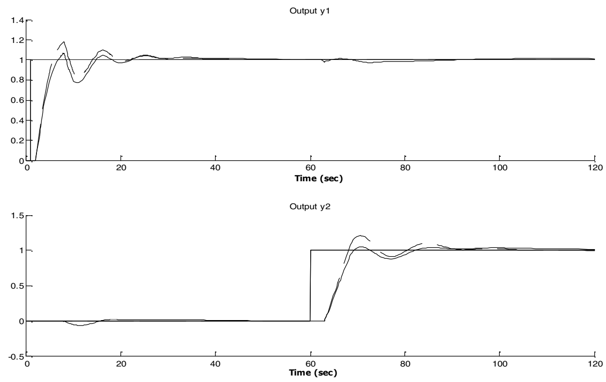

Figure 4: Step response of Wood-Berry column, with the PI decoupler (41) and proportional controllers corresponding to margins A_{m1}=5 and A_{m2}=3. In dashed line, PI + ideal decoupler from [9]

Note that the two off-diagonal elements of equation (41) are PI controllers plus delay time. But it could be considered the delay time in the model reduction, obtaining the integral decoupling matrix without delays (42). Consequently, a good frequency response approximation of k_{12} and k_{21} is not possible, the loop interactions increase and the step response of the closed-loop system deteriorates, compare Fig. 5 with Fig. 4. This result is similar to the obtained by Wang et. al. [7] using a centralized PID controller, that it is shown in same Fig. 5, in dashed line.

$$D_{WB2}^{PI}(s) \cong \begin{pmatrix} \frac{s+0.13}{0.81s} & -\frac{s+0.343}{2.16s} \\ \frac{-s+0.304}{1.57s} & -\frac{s+0.165}{1.57s} \end{pmatrix} \quad (42)$$

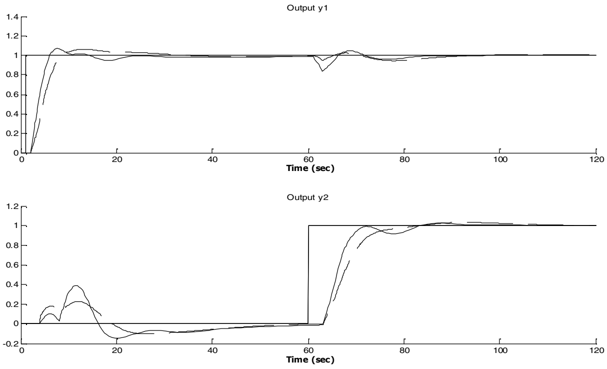

Figure 5: Step response of Wood-Berry column, with the PI decoupler without delays (42) and proportional controllers corresponding to margins A_{m1}=5 and A_{m2}=3. In dashed line, centralized PID [7]

3.2. The quadruple-tank process

The quadruple-tank process [1, 17] has been used to illustrate many issues in multivariable control. Physical modelling leads to the following linearized transfer matrix linking the voltages applied to the pumps with the levels

$$G(s) = \begin{pmatrix} \frac{\gamma_1 \alpha_1}{T_1 s+1} & \frac{(1-\gamma_2)\alpha_1}{(T_1 s+1)(T_3 s+1)} \\ \frac{(1-\gamma_1)\alpha_2}{(T_2 s+1)(T_4 s+1)} & \frac{\gamma_2 \alpha_2}{T_2 s+1} \end{pmatrix} \quad (43)$$

where $T_i>0$, $\alpha_i>0$, $0\leq\gamma_i\leq 1$, $\forall i$. The system is non-minimum phase for $0<\gamma_1+\gamma_2<1$ and minimum phase for $1<\gamma_1+\gamma_2<2$. Moreover, its RGA is $\gamma_1\gamma_2/(\gamma_1+\gamma_2-1)$. Next design has been done with the values of a system built at the University of Newcastle [12]: time constants in minutes (T_1=1, T_2=1.5, T_3=0.5, T_4=0.5), α_1=3.7, α_2=4.7 and also selecting γ_1=0.5 and γ_2=0.3 in order to test a non-minimum phase process (RHP zero in z≅1.06) with medium interaction (RGA≅-0.75).

We choose the two open-loop transfer functions $l_1(s)$ y $l_2(s)$ following the second row of table 1 with $\tau_1 = \tau_2 = 0$ and z=1.06. The approximated PI decoupling matrix is obtained for the quadruple-tank process:

$$D_{QT}^{PI}(s) = \begin{pmatrix} -\frac{s+1.09}{5.62s} & \frac{s+1.58}{4.15s} \\ \frac{s+4.01}{11.56s} & -\frac{s+0.74}{2.88s} \end{pmatrix} \quad (44)$$

In comparison with the decoupler (41) designed for the Wood and Berry column, the decoupler (44) does not need delay times, but it can be remarked that only good frequency response approximation of k_{11} and k_{22} is possible, while the frequency response approximation of k_{12} and k_{21} is only good at low frequencies. A better approximation would require second order denominator in the two off-diagonal elements of (44).

The tuning parameters are firstly $k_2=k_1\cong 0.48$, corresponding to $\delta_2=\delta_1=0.4$ in (26). The resultant step response of the closed-loop system is shown in Fig. 6. Both loops show the same response (solid lines), close to the specified by the transfer function (27). However, if the parameter k_1 is duplicated, the step response of loop 1 (dashed line) changes significantly while the step response of loop 2 changes slightly. Therefore, the decoupling from loop 1 to loop 2 is very satisfactory.

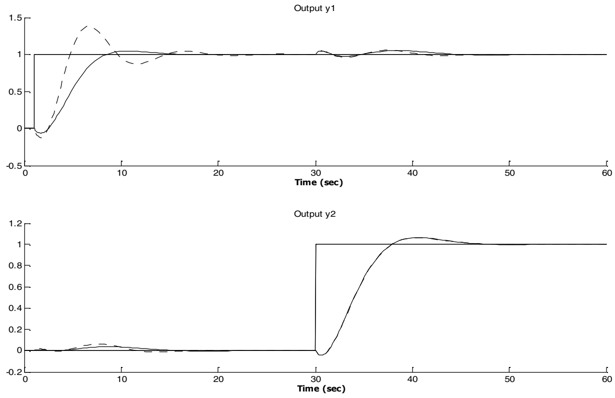

Figure 6: Step response of the quadruple-tank process, with the PI decoupling (44) and $k_2=k_1\cong 0.48$ (solid lines) and $k_1=2k_2\cong 0.96$ (dashed lines).

The Fig. 7 shows the closed-system response to sudden changes in the two control signals. Both loops react suitable to these changes, returning the output signals to the set points. Also note that, as in Fig. 6, the parameter k_1 can be used to change significantly the response of loop 1 (dashed line) while the response of loop 2 remains almost unchangeable.

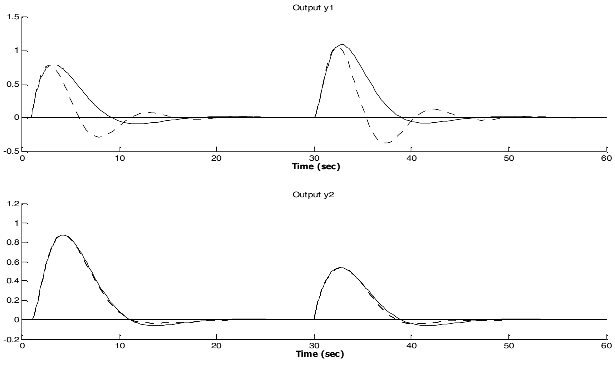

Figure 7: Response of the quadruple-tank process to sudden changes in the two control signals, with the PI decoupling (44) and $k_2=k_1\cong 0.48$ (solid lines) and $k_1=2k_2\cong 0.96$ (dashed lines).

3.3. The Twin Rotor MIMO System

The Twin Rotor from Feedback Ltd. (www.fdbk.com) is a non-linear multivariable system with slight static cross-coupling but with significant dynamic cross-coupling. The two controlled variables are the yaw angle (horizontal position) and the pitch angle (vertical position) and the two manipulated variables are the voltages applied to the rotors. The main rotor shows a very oscillatory behaviour and the tail rotor presents an integral behaviour. Both rotors show inherently difficult control problems. A linearized transfer matrix for this system at horizontal position equal to zero is given by

$$\begin{pmatrix} \dfrac{34.88}{1.43s^3+1.07s^2+2.05s+1.40} & \dfrac{13.36s}{0.38\,s^3+1.02s^2+0.59s+1.40} \\ \dfrac{19.88s+13.50}{\left(8.43s^2+7.26s+1\right)s} & \dfrac{342.90}{\left(2.24s^2+6.21s+1\right)s} \end{pmatrix} \quad (45)$$

The system has a RHP zero in $z \cong 6.38$, but it is not necessary to take it in consideration because it is very far from the process dominant poles. Therefore, the PID decoupling matrix (46) is obtained for the Twin Rotor when using $k_1=0.05$, corresponding to $T_1=20$ in (22), and using $z_2=1/20$ and $k_2=0.2$, corresponding to $\delta_2=1$ and $\omega_{n2}=0.1$ in equation (30).

The designed controller is shown in the following equation:

$$D_{TR}^{PID}(s) = \begin{pmatrix} \dfrac{s^2+1.82s+1.24}{30.79s} & -\dfrac{s^2+0.037s+2.66\ 10^{-11}}{656s} \\ -\dfrac{s^2+1.36s+0.72}{454s} & \dfrac{s^2+0.206s+0.0079}{54.1s} \end{pmatrix} \quad (46)$$

$$\cong \begin{pmatrix} \dfrac{s^2+1.82s+1.24}{30.79s} & 0 \\ -\dfrac{s^2+1.36s+0.72}{454s} & \dfrac{s^2+0.206s+0.0079}{54.1s} \end{pmatrix}$$

Fig. 8 shows the step response of the closed-loop system.

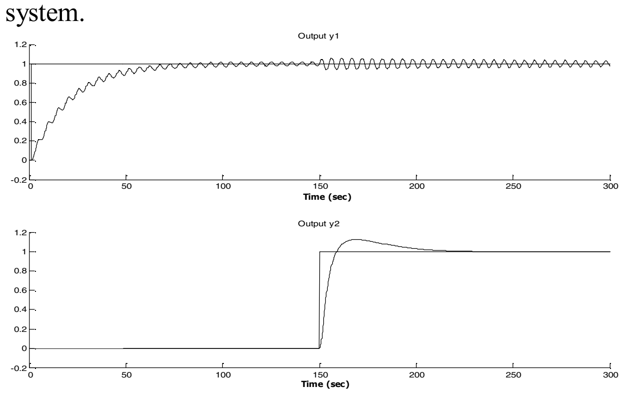

Figure 8: Step response of Twin Rotor, with the PID decoupler (46) and proportional controllers k_1=0.05 and k_2=0.2.

Both loops show a response very close to the specified by the transfer functions (22) and (29) respectively. In addition, the coupling from loop 1 to loop 2 has almost disappeared. This benefit is motivated by the element (2,1) in the decoupler (46), while the effect of the element (1,2) is negligible, resulting a partial decoupling. A PI decoupler has also been tested but it is not advisable due to the oscillatory modes of this system.

4. Conclusions

A new methodology for designing 2×2 PID controllers has been presented. First, a decoupler with integral action is designed. Next, the decoupler is approximated by a PID network including delay. In this way, the resulting full matrix controller can be easily implemented in commercial distributed control systems.

The methodology has been illustrated with three representative examples, found in multivariable literature, obtaining good results. The most relevant aspect of the new methodology has been shown in simulation: a PID controller network is used to compensate the structural part of the process and two gains (k_1 and k_2) act as degrees of freedom in order to modify the behaviour of the two loops almost independently.

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