

PID CONTROL DESIGN WITH GUARANTEED STABILITY

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Abstract: This paper presents a Matlab GUI to design PID controllers with guaranteed stability. The GUI shows the stability region in the respective parameter plane K_P - K_I or K_P - K_D and the boundary curves for frequency response requirements (phase margin, gain margin, maximum sensitivity and maximum complementary sensitivity). Combining several requirements the tool splits the stability region into zones and the user can explore them in order to tune the controller satisfying those requirements. The GUI can be also used in lectures about stability, robust PID control, PID design in the frequency domain, PID Loop Shaping, and it can be very useful for students to get insight into tuning PID controllers. *Copyright* © 2006 IFAC.

Keywords: computer aided design, PID control, robust control.

1. INTRODUCTION

In the last years some special design methods for PID controllers based on loop shaping have been proposed. They can be used for any arbitrary order irrational or non-minimum phase transfer functions with dead-time (Åström and Hägglund, 2005; Čech and Schlegel, 2005; Dormido and Morilla, 2004; Morilla and Dormido, 2000). These methods use robustness conditions, expressed in terms of maximum sensitivity M_s and maximum complementary sensitivity M_t , or stability margins expressed in terms of phase margin ϕ_m and gain margin A_m . These requirements involve that the Nyquist curve of the loop transfer function should avoid the respective M_s or M_t circle, it should intersect the third or fourth quadrant of the unit circle or it should intersect the negative real axis far away from the critical point -1, as Figure 1 shows.

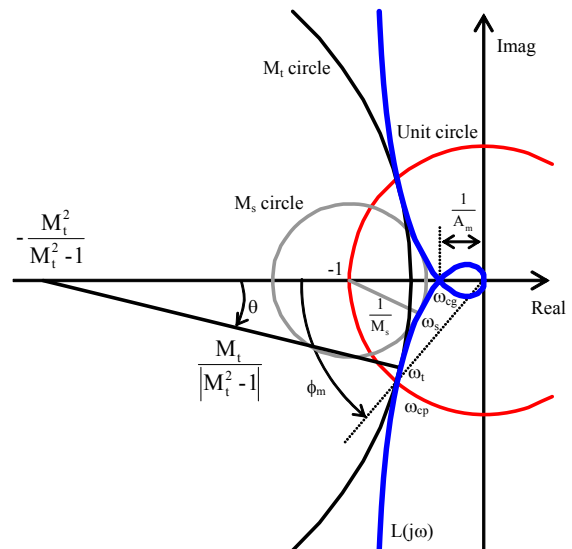


Fig. 1. Example of Nyquist plot of the loop transfer function $L(s)$ and its features.

These requirements give a set of admissible values of the controller parameters, describing the boundaries of robustness region in the parameter plane K_P - K_I or K_P - K_D of the PID controller (Shafiei and Shenton, 1997; Schlegel and Mertl, 2004). Most of these methods consider only positive control gains and cannot guarantee the stability of the closed loop system. Neither they cannot assure that the frequency response have the requirements specified.

The main aim of this paper is to show how the PID controller can be designed based on frequency response requirements assuring stability of the closed loop system. In Section 2, the stability regions for five types of PID controllers are presented. The boundary curves for frequency response requirements are given in Section 3. The PID graphic user interface that put in practice these methodologies is described in Section 4. Conclusions are presented in Section 5.

2. STABILITY REGIONS

As it is well known, stability is a primary requirement on a feedback system. Therefore, dealing with PID control it is important to know the set of controller parameters (it will be called the stability region) so that given a point of this region the closed loop system is stable. Moreover, it turns out that much insight into PID control can be obtained by analyzing the stability regions (Åström and Hägglund, 2000).

Consider the feedback control system shown in Figure 2, in which the process to be controlled is described by the transfer function

$$P(s) = \frac{B(s)}{A(s)} \quad (1)$$

Let the controller $C(s)$ of the PID type:

$$C(s) = K_p + \frac{K_i}{s} + K_D s \quad (2)$$

where K_p , K_i , K_D , are the proportional, integral and derivative gains of the controller, respectively. The characteristic equation of the closed-loop system is given by

$$f(s; K_p, K_i, K_D) = s A(s) + s^2 K_D B(s) + s K_p B(s) + K_i B(s) = 0 \quad (3)$$

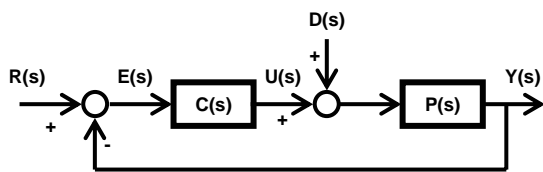


Fig. 2. Feedback control system.

The stability region of this family is a subset of R^3 because there are three parameters. When $A(s)$ and $B(s)$ are polynomials the characteristic equation (3) is a family of polynomials called polytope of polynomials (Barmish, 1988) and, as it is well known, it can be formed by several disjoint sets.

In order to obtain stability regions for polytopes some results can be found. The most relevant and useful was shown by Ackermann (Ackermann, 1980). This result allows obtain the stability region in terms of the parametric space, so that necessary and sufficient conditions can be established and the stability region can be calculated in the plane with a low computational cost. With three parameters, a sweeping along one of them is necessary, growing the computational cost.

However, there are many other applications. So, it can be used to calculate the maximum stability box or the stability region for predictive controllers with two parameters (Mañoso, 1996).

In order to design PID controllers with guaranteed stability this paper considers the five R^2 (plane) particular cases, shown in Table 1 joint with their characteristic equation:

PI, when $K_D = 0$.

PD, when $K_I = 0$.

PID α , when the ratio between the derivative and integral time constants $\alpha = T_D / T_I$ is fixed, Note that this is equivalent to fix $K_D = \alpha \frac{K_p^2}{K_i}$

PIDK $_D$, when K_D is fixed.

PIDK $_I$, when K_I is fixed.

Table 1: Stability regions on the parameter plane

Control	Plane	Characteristic equation
PI	K_p - K_i	$f_{PI}(s; K_p, K_i) = s A(s) + s K_p B(s) + K_i B(s) = 0$
PD	K_p - K_D	$f_{PD}(s; K_p, K_D) = A(s) + s K_D B(s) + K_p B(s) = 0$
PID α	K_p - K_i	$f_{\alpha}(s; K_p, K_i) = s K_i A(s) + s^2 K_p^2 \alpha B(s) + s K_p K_i B(s) + K_i^2 B(s) = 0$
PIDK $_D$	K_p - K_i	$f_{KD}(s; K_p, K_i) = s A_D(s) + s K_p B_D(s) + K_i B_D(s) = 0$
PIDK $_I$	K_p - K_D	$f_{KI}(s; K_p, K_D) = A_I(s) + s K_D B_I(s) + K_p B_I(s) = 0$

where:

$$A_D(s) = A(s) + s K_D B(s) \text{ and } B_D(s) = B(s)$$

$$A_I(s) = s A(s) + K_i B(s) \text{ and } B_I(s) = s B(s)$$

The notation used in Table 1 allows that the PIDK $_D$ control includes the PI control as a particular case when $K_D=0$, and the PIDK $_I$ control includes the PD control as a particular case when $K_I=0$. In order to do this, it must be noted that the contribution of the

control parameters, K_D and K_I respectively, are involved into the process transfer function.

Figure 3 shows two examples of stability regions for a non minimum phase process describe by the transfer function

$$P(s) = \frac{-10(s-2)}{(s+1)(s+5)(s+10)} = \frac{-10s+20}{s^3+16s^2+65s+50} \quad (4)$$

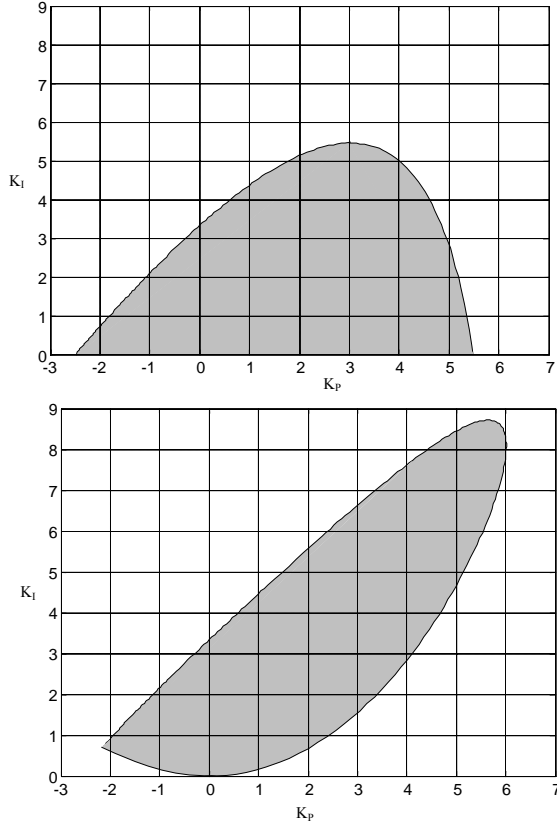


Fig. 3. Stability region of the non minimum phase process (4). (a) With PI control. (b) With PID control, using the ratio proposed by Ziegler and Nichols $\alpha=0.25$.

The process transfer function in equation (1) does not include dead-time in order to determine the stability regions accurately.

3. BOUNDARY CURVES

In this paper, the four requirements depicted in Figure 1 are considered. As it is well known, these performances could be reached by loop shaping, doing that the target point B lies on the Nyquist plot $L(j\omega)=C(j\omega)P(j\omega)$ (Åström and Hägglund, 2005; Dormido and Morilla, 2004). Table 2 gives the target points B corresponding to the four requirements, where r_B and ϕ_B are the magnitude and the phase of point B with respect to the origin and to the negative real axis. R and c are the centre and the radius of the respective circle. θ is the angle where the Nyquist plot contacts the respective circle.

These requirements give a subset of R^3 , all possible trios of gains (K_P, K_I, K_D) verifying the expressions

$$K_P = \frac{r_B \cos(\phi_B - \phi(\omega))}{r(\omega)} \quad (5)$$

$$\omega K_D - \frac{K_I}{\omega} = \frac{r_B \sin(\phi_B - \phi(\omega))}{r(\omega)} \quad (6)$$

where $r(\omega)$ and $\phi(\omega)$ are the magnitude and the phase of $P(j\omega)$ with respect to the origin and to the negative real axis. Thus, one point A ($r(\omega_d), \phi(\omega_d)$) of the process Nyquist plot

$$P(j\omega) = r(\omega) e^{j(\phi(\omega)-180)} \quad (7)$$

is mapping to the target point B (r_B, ϕ_B) of the loop Nyquist plot

$$\begin{aligned} L(j\omega) &= C(j\omega)P(j\omega) = \\ &= \left(K_P - j \frac{K_I}{\omega} + j K_D \omega \right) P(j\omega) \end{aligned} \quad (8)$$

where ω_d is called the design frequency.

Table 2 : Target points for the four requirements

Requirement	Target point
PM (ϕ_m)	$\phi_B = \phi_m$; $r_B = 1$
GM (A_m)	$\phi_B = 0^\circ$; $r_B = \frac{1}{A_m}$
Ms (M_s, θ) $c = -1$ $R = \frac{1}{M_s}$	$\phi_B = \text{atan} \frac{R \sin \theta}{-c - R \cos \theta}$ $r_B = \frac{R \sin \theta}{\text{sen} \phi_B}$
Mt (M_t, θ) $c = -\frac{M_t^2}{M_t^2 - 1}$ $R = \frac{M_t}{ M_t^2 - 1 }$	

The expressions (5) and (6) have been particularized in Table 3 when additional conditions, mentioned in section 2 are imposed to the controller gains. In this way a subset of admissible values of the controller parameters can be shown as a boundary curve, in the respective parameter plane K_P - K_I or K_P - K_D . The second row of Table 2 includes the PI control as particular case when $K_D=0$, and the third row includes the PD control as particular case when $K_I=0$.

Even using a reduced range of frequencies, it is usual that only a bit of the boundary curve lies inside the corresponding stability region. Only this bit of the boundary curve is interesting for find the control parameters. The Figure 4 shows an example of boundary curve obtained for the phase margin $\phi_m=60^\circ$. This curve splits the stability region into two zones: I ($\phi_m > 60^\circ$) and II ($\phi_m < 60^\circ$).

With these information is easy to tune the controller. A point of the boundary curve could be select to get $\phi_m=60^\circ$, a point of the zone I to get $\phi_m>60^\circ$ or a point of the zone II to get $\phi_m<60^\circ$. It is suitable to choose the point with the maximum K_I (Dormido and Morilla, 2004). But also it is not suitable to choose any point near the boundary of stability.

Table 3 : Boundary curves for PID control

Control	Boundary curves
PID α	$K_p = \frac{r_B \cos(\phi_B - \phi(\omega))}{r(\omega)}$ $K_I^2 r(\omega)^2 + K_I \omega r_B r(\omega) \sin(\phi_B - \phi(\omega)) - \omega^2 \alpha r_B^2 \cos^2(\phi_B - \phi(\omega)) = 0$
PIDK $_D$	$K_p = \frac{r_B \cos(\phi_B - \phi(\omega))}{r(\omega)}$ $K_I = \omega^2 K_D - \frac{\omega r_B \sin(\phi_B - \phi(\omega))}{r(\omega)}$
PIDK $_I$	$K_p = \frac{r_B \cos(\phi_B - \phi(\omega))}{r(\omega)}$ $K_D = \frac{K_I}{\omega^2} + \frac{r_B \sin(\phi_B - \phi(\omega))}{\omega r(\omega)}$

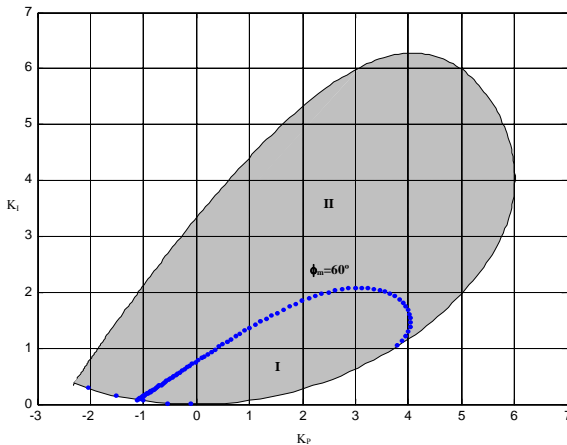


Fig. 4. Stability region and boundary curve for PID control of the non minimum phase process (4). With $\alpha=0.1$ and $\phi_m=60^\circ$.

Moreover it is normal that some points of the boundary curve inside the respective stability region were invalid solutions, because with these controller parameters the Nyquist plot has multiple crossing. The Figure 5 shows an example of boundary curve with some invalid points (the points outside the loop). This curve splits the stability region into two zones: I ($A_m > 4$) and II ($A_m < 4$).

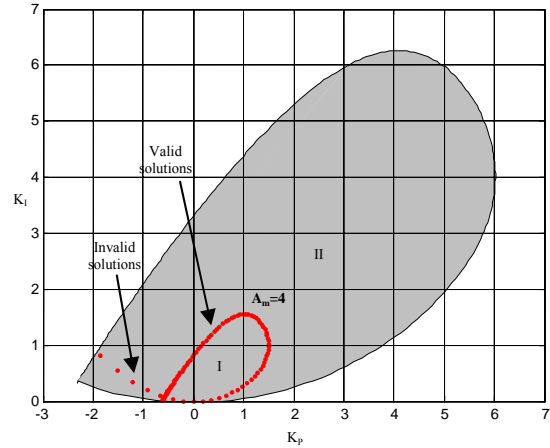


Fig. 5. Stability region and boundary curve for PID control of the non minimum phase process (4). With $\alpha=0.1$ and $A_m=4$.

Combining two values of the same requirement or different requirements it is possible to get a lot of information. The Figure 6 is an example where four boundary curves split the stability region into ten zones:

- I - $\phi_m \geq 60^\circ$ $A_m \leq 4$
- II - $\phi_m \geq 60^\circ$ $A_m \geq 4$
- III - $\phi_m \geq 60^\circ$ $2 \leq A_m \leq 4$
- IV - $\phi_m \geq 60^\circ$ $A_m \leq 2$
- V - $\phi_m \geq 40^\circ$ $A_m \leq 2$
- VI - $40^\circ \leq \phi_m \leq 60^\circ$ $A_m \leq 4$
- VII - $40^\circ \leq \phi_m \leq 60^\circ$ $A_m \leq 4$
- VIII - $40^\circ \leq \phi_m \leq 60^\circ$ $A_m \leq 2$
- IX - $\phi_m \leq 40^\circ$ $A_m \geq 2$
- X - $\phi_m \leq 40^\circ$ $A_m \leq 2$

But also there are five important points, the five points where the boundary curves cross over:

- a - $\phi_m = 60^\circ$ $A_m = 4$
- b - $\phi_m = 60^\circ$ $A_m = 4$
- c - $\phi_m = 60^\circ$ $A_m = 2$
- d - $\phi_m = 40^\circ$ $A_m = 2$
- e - $\phi_m = 40^\circ$ $A_m = 2$

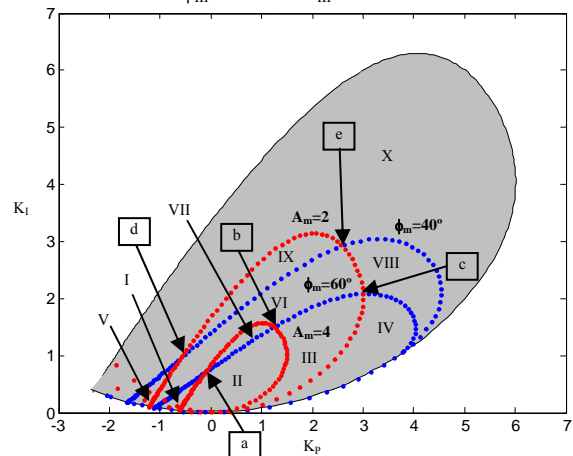


Fig. 6. Stability region and boundary curves for PID control of the non minimum phase process (4). With $\alpha=0.1$, $40^\circ \leq \phi_m \leq 60^\circ$, and $2 \leq A_m \leq 4$.

When the requirements M_s and M_t are used, it is suitable to determine the boundary curve for a range of θ , for example between 5° and 45° , instead of a single value. Moreover the tangent condition in the target point must be verified (Dormido and Morilla, 2004). This condition is summarized in the following expression:

$$r_B r'_B(\omega) + c (r'_B(\omega) \cos \phi_B - r_B \phi'_B(\omega) \sin \phi_B) = 0 \quad (9)$$

where $r'_B(\omega)$ and $\phi'_B(\omega)$ represent the value of the derivative of the magnitude and phase of $L(j\omega)$ in the point B respectively.

Therefore obtaining the boundary curves for M_s and M_t need more calculations than the curves for ϕ_m and A_m . The Figure 7 is an example where two boundary curves for maxima of sensitivity split the stability region into three zones:

- | | | | |
|-----|---|----------------|----------------|
| I | - | $M_s \leq 1.8$ | $M_t \leq 1.6$ |
| II | - | $M_s \geq 1.8$ | $M_t \leq 1.6$ |
| III | - | $M_s \geq 1.8$ | $M_t \geq 1.6$ |

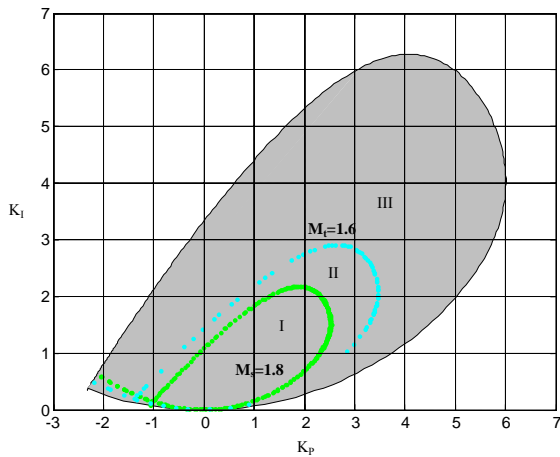


Fig. 8. Stability region and boundary curves for PID control of the non minimum phase process (4). With $\alpha=0.1$, $M_s=1.8$, and $M_t=1.6$.

4. THE PID GUI

A PID graphic user interface (GUI) has been developed in Matlab. The main window is shown in Figure 9. It is designed to provide simultaneously enough information (the stability region, the boundary curves, the time and frequency responses) and to update it quickly when the user makes changes in the interface.

The transfer function without dead-time (numerator and denominator) describing the process is on the upper left corner. A pop-up menu with the five kinds of designs (PI, PD, PID α , PIDK $_D$ and PIDK $_I$) is on the upper left corner. The radio-buttons for select the design requirements are on the middle. The requirements are given by single values or by range between two values in four edit fields.

The most valuable information is shown in the parameter plane, K_p - K_I or K_p - K_D depending on the chosen design. The set of yellow lines delimit the stability region, the user can to hide it or to display it using the radio-button on top of the parameter plane. The red, blue, green and cyan points represent boundary curves for phase margin, gain margin, maximum sensitivity and maximum complementary sensitivity respectively. To hide or to display these curves the user must use the requirements radio-buttons.

A black point can be dragged with the mouse inside the parameter plane in order to test different control gains. When this point is released; the three edit fields show the control gains (K_p , K_I and K_D), the Nyquist plot and the frequency response features are updated. A new simulation is also performed and the new time closed loop response is added to the graphics.

In Figure 9 a PID α controller with $\alpha=0.1$ has been tuned for the non minimum phase process (4). Four boundary curves has been calculated for single values of the specifications ($\phi_m = 60^\circ$, $A_m = 3$, $M_s = 1.8$, $M_t = 1.6$) and five values of θ between 10° and 40° . The black point has been dragged to the zone where $\phi_m < 60^\circ$, $A_m < 3$, $M_s < 1.8$ y $M_t < 1.6$. These constraints shape the loop Nyquist plot to get ($\phi_m = 55.6^\circ$, $A_m = 2.7$, $M_s = 1.7$, $M_t = 1.1$) showing that the design was possible.

Note that other important information, the frequencies where the loop performances happen, is presented in Figure 9. In this case, the frequencies are in the following order: $\omega_{cp} \cong 0.8$, $\omega_t \cong 0.9$, $\omega_s \cong 2.0$, $\omega_{cg} \cong 3.2$. The main windows show also the slope of the loop Nyquist plot in each of these frequencies and the angles where the plot contacts the circles M_s and M_t . All these information is very important to decide if the Nyquist plot has a good shape (Åström and Hägglund, 2005).

With the control gains selected $K_p \cong 2.0$, $K_I \cong 2.0$ and $K_D \cong 0.2$, the time response of the closed loop is good. The output and the set point signal are shown on the left. The control signal is shown on the right. Comparing these graphics with the Nyquist plot is a good way to develop intuition for the relations between the time and frequency responses.

The toolbar of the main window includes a menu, where user can load and save designs, import a process from workspace, export a controller to workspace, set the simulation parameters and choice several aspects of designs.

5. CONCLUSIONS

The Matlab GUI described in this paper has been developed for teaching activities in two Spanish universities, improving previous tools of the authors. It will be available on <http://www.uco.es/~in2vasef/>. This paper presents the fundamentals of this GUI but

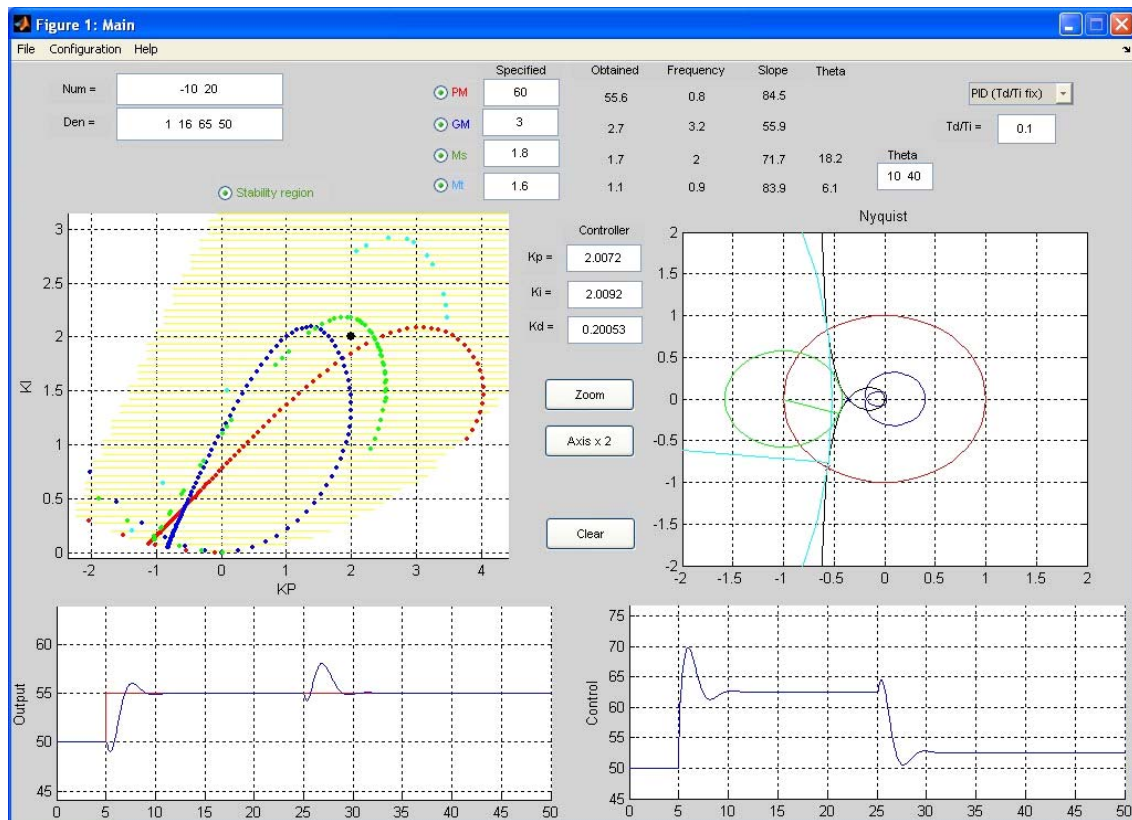


Fig. 9. Main window of the Matlab GUI to design PID controller with guaranteed stability.

it also show some of the general benefits of using a tool like this to understand concepts about stability, robustness and stability margins in PID control loop. A non minimum phase process has been used in the examples but the GUI has been tested with a great number of transfer functions.

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