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TUNING DECENTRALIZED PID CONTROLLERS FOR MIMO SYSTEMS WITH DECOUPLERS

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Abstract: In this paper, a method for controlling multivariable processes is presented. The controller design is divided into two parts: firstly, a decoupling matrix is designed in order to minimize the interaction effects. Then, the controller design is obtained for the process + decoupler block. For this purpose, an iterative numeric algorithm, proposed by same authors, is used. The aim is to meet the design specifications for each loop independently. This sequential design method for multivariable decoupling and multiloop PID controller is applied to several examples from literature. Decentralized PID controller design, specifications analysis and time response simulations has been made using the TITO tool, a set of m functions written in Matlab. It can be obtained in web page http://www.uco.es/~in2vasef. *Copyrigth © 2002 IFAC.*

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1. INTRODUCTION

Most of industrial processes are multivariable in essence (Shinskey, 1995). For such systems, loop interactions can arise and cause difficulties in feedback controller design. To solve this problem there exist several alternatives that can be centralized multivariable control (LQ, LQR and LQG, robust control, predictive multivariable control, fuzzy control,...) or decentralized control. This last is the most broadly used in the industrial environment. With decentralized techniques, a multivariable system with n inputs and n output variables is treated as n monovariable systems. The sacrifice that supposes the invariable performance deterioration of a decentralized control structure when it is compared with a full multivariable control strategy is compensated with certain advantages as design and hardware simplicity or easiness of use.

Some of decentralized methods found in literature could be classified under the following topics, showing the interest that it has raised in the last years:

First group bases their design on some SISO method. These formulas indicate the direction in which the PID parameters have to be detuned to compensate the interaction effects when all loops are closed. Method of Shinskey (1995) and BLT method (Luyben, 1992), one of most cited in literature, are included in this group. However, with these methods it is difficult to establish a priori design specifications in all loops and they can be rather considered of trial and error.

A second group includes works that look for critical gains of the system in order to tuning the PID controllers. This gain can be obtained by means of proportional controllers (Niederlinski, 1971), or by means of relay method (Zhuang and Atherton, 1994), (Halevi et al, 1997), (Toh and Devanathan, 1993), (Shiu and Hwang, 1998). Not all these methods require a complete model of the systems.

Lastly, methods included in a third group use the whole transfer function matrix taking into account the interaction effects, and the controllers are obtained by means of analytic, numeric or graphics methods. Some methods (Wang *et al*, 2000) use optimization algorithms to obtain controllers, other (Zhang *et al*, 2000) uses pole placement techniques, and some (Ho et al, 1995) provides on-line tuning formulas. Method described in section 3 (Vázquez *et al*, 1999) also belongs to this group.

But sometimes, interaction between variables or system dynamics can impede the application of one of previous decentralized methods, and design specifications are not met. In this case, and before using some centralized method, there is an hybrid alternative consisting of decomposing the design problem into two parts: firstly, decoupling the system in order to minimize interaction or to make the system diagonal dominant; then, design the controllers using some decentralized method. The final control system will be the product of the decoupling and the controller matrixes. The decouplers, together with single-loop controllers, constitute the multivariable controller. This alternative could be used as another design option. But not all decentralized design methodologies are prepared to do that. Section 2 described how decoupling can be done, and section 3 how a decentralized design technique (Vázquez *et al*, 1999) fulfill this purpose.

2. DECOUPLING DESIGN

The design of a decentralized control system with a decoupling matrix can be done combining a diagonal controller $K_d(s)$ with a block compensator D(s), so that the controller manipulates the variable u'_i instead of the u_i, as can be appreciated in figure 1, for the 2 x 2 case. With this configuration the controller see the process as a set of n completely independent processes or with the interaction minimized.

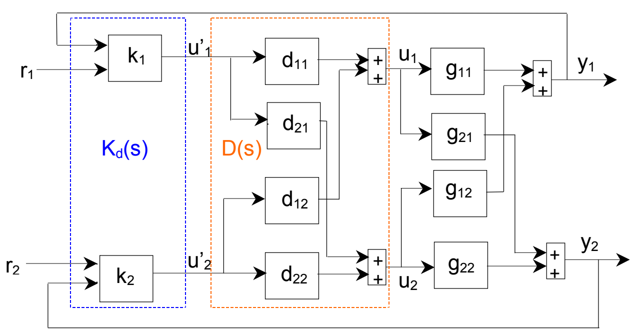

Fig. 1 : General 2 x 2 system with decouplers and single-loop controllers

The essence of decoupling is the imposition of a calculation net that cancels the existent process interaction, allowing the independent control of the loops. In decentralized design, the question is not to eliminate interaction, but to take it into account. A multivariable system may still experience interactions and responds poorly. The objective in decoupling is to compensate for the effect of interactions brought about by cross coupling of the process variables.

In literature there are different decoupling methods: Lineal decoupling (Desphande, 1989) is most extended method. In this, the decoupling matrix try to eliminate interactions from all loops, obtaining following elements for a 2 x 2 system.

$$d_{11}=1,\, d_{12}(s) = -\frac{g_{12}(s)}{g_{11}(s)},\, d_{21}(s) = -\frac{g_{21}(s)}{g_{22}(s)},\, d_{22}=1 \quad (1)$$

Implementation of this decoupling matrix has some problems: What happens if numerator has bigger order than denominator?, what if delays exist?, and what if these delays only appear in the denominator?. The different works propose some solutions like Pade approximations for delays or steady state decoupling as a first option to prevent this problems. Other solutions consist of applying partial decoupling, setting d_{12} or d_{21} null. This fact avoid problems originated in one loop reach the other.

There are some other methods: one of them looks for diagonal dominance in the system as ALIGN algorithm designed by McFarlane and Kouvaritakis in 1974 and described in (Maciejowski, 1989). Other design by means of singular value decomposition (Desphande, 1989) or by means of inverse decoupling (Wade, 1997) that produces the process input signals by combining one controller output with the other process input signals.

Even if decouplers are incorporated, the interaction effects cannot be completely eliminated because of model mismatch. Then, single-loop controllers cannot be tuned independently, and a sequential tuning algorithm that takes interactions into account should be used. Next section describes one of these algorithms.

3. MIMO DESIGN METHODOLOGY

Analysis of decentralized methods described in section 1 show several interesting points:

- Obtaining different design specifications for each loop implies solution methods based on some kind of iteration. If designs are carried out loop-by-loop, tuning one of them can detune the others.
- In spite of abundance of methods, there are no simple and general solutions for the multivariable tuning problem. Or methods are used by optimization (Wang *et al*, 2000) that do not guarantee an appropriate solution and they require several iteration processes, or the solutions are too particular, for concrete models (Ho *et al*, 1996) or too simple (Shinskey, 1996), (Luyben, 1992) that only get a first approach to the problem.

In (Vázquez *et al*, 1999), a method of tuning PID controllers for systems with decentralized control is presented. Its fundamental characteristics can be summarized in following points:

- It is a generic method for n x n systems with decentralized control.
- It is a method based on successive SISO tuning that does not suppose any additional constraint to obtained controllers (except for their decentralized structure) neither to the transfer functions matrix. This matrix can include the transfer functions of only the process or also including the decoupling net. Present work exploits this characteristic.
- The SISO methodology integrated in the algorithm has certain imposed limitations: it should allow the controller be designed from a frequential description of the process (their frequency response). And it should quantify the specifications achievement by means of a quality index, J. This index controls the iteration evolution of the tuning algorithm.
- The method allows a decoupling matrix to be included between the plant and the controllers.
- The design methodology is divided into two phases that are carried out in a sequential and combined way: the structural decomposition and the controller design. These two phases are described next.

3.1. System structural decomposition

This phase consists of the decomposition of a n x n multivariable system into n SISO systems. To solve this problem the structural decomposition, introduced by Zhu (1996), is used. Let be a n input system controlled by means of a decentralized control strategy. Now, n-1 loops have been closed by means of n-1 controllers. The process 'seen' from the free input to the free output is needed. The scheme corresponds to the figure 2, where loop i has not yet been closed (the pairing problem has been already solved and input-output pairs are in the diagonal).

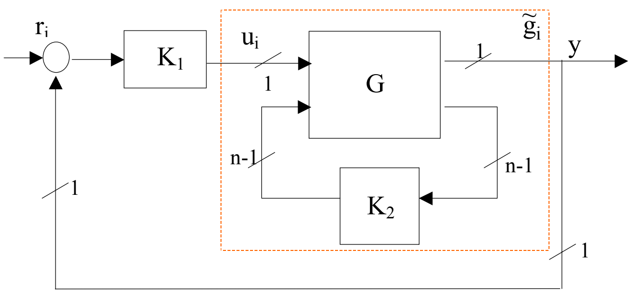

Fig. 2: Structural Decomposition

In the structural decomposition scheme, K_1 is the controller that closes the loop between input i and output i. The set of n-1 controllers, whose loops are already been closed, is K_2. This way, the controller matrix is:

$$K = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix} \quad (2)$$

The process element g_{ii} has its feedback loop open. The rest of elements of its same row are called G_{12}. The rest of elements of its same column are called G_{21}. And the rest of elements of G are called G_{22}. Then, the process matrix G can be written as:

$$G(s) = \begin{pmatrix} g_{ii}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{pmatrix} \quad (3)$$

The process seen from input i to output i when the other loops has been closed is called $\tilde{g}_i$ and it is obtained from

$$\tilde{g}_i = g_{ii} - G_{12}K_2(I + G_{22}K_2)^{-1}G_{21} \quad (4)$$

where, using the above notation, g_{ii} is an element of G, that is, a SISO transfer function, G_{12} a transfer function row vector of dimension 1 x (n-1), G_{21} a transfer function column vector of dimension (n-1) x 1, K_2 is a diagonal square matrix of dimension (n-1) and I is the identity matrix of order n-1. In the case of 2 x 2 processes, elements G_{12} and G_{21} are also individual SISO transfer function elements of G, and the calculation of $\tilde{g}_i$ do not imply matrix operations.

Opposite to standard transfer functions, the $\tilde{g}_i$ are not an intrinsic property of the system, and they depend on the designed controllers, as can be observed in equation (4). Possibly, they have not a representation by means of Laplace transformation.

The structural decomposition implies some important considerations for the decentralized control: some global system properties, as interaction, stability, integrity, etc, could be deduced from the properties of the obtained n SISO subsystems.

Also, no supposition about G has been made, and then, structural decomposition could be applied to every transfer functions matrix. Applying Nyquist theorem to each individual loop obtained with structural decomposition, stability criterion is more interesting from the point of view of applicability to decentralized multivariable systems. Then, the n SISO systems stability implies MIMO stability.

Theorem 1: Supposing that individual elements of G(s) and their SISO independent subsystems do not have poles in the right hand plane, the system with decentralized control is stable if and only if the Nyquist contour of the equivalent open loop transfer function $k_i\tilde{g}_i$ does not encircle point (-1,0), i.e.

$$N(-1, k_i\tilde{g}_i) = 0 \quad \forall i \quad (5)$$

The structural direct Nyquist arrays (SDNA) are the representation of the n direct Nyquist diagrams of $N(-1, k_i\tilde{g}_i) = 0 \quad \forall i$.

3.2 Controllers design

The proposed is a frequency-based method and stability measures as phase and gain margins could be used. But most of existent methods requires a model for obtaining the design parameters. Carrying out the structural decomposition, the resulting transfer function $\tilde{g}_i$ could be not a rational function, possibly due to the inclusion of delays in model systems. In order to solve this problem a frequential representation of this transfer function $\tilde{g}_i$ it is used, i.e. a frequency response array. But this take implicit another problem: what SISO design method to use? The particular solution has been the monovariable methodology described in (Morilla and Dormido, 2000). This methodology supposes, and in fact requires, a model with the description of the system dynamics. However it leaves open the possibility to be adapted to solve the problems outlined previously: it can be modified allowing the controllers to be designed from an array with the frequential representation of the frequency response and also, the obtained results can be compared with the design specifications to present an indicative of quality, indispensable for the proposed solution search method. These two phases cannot be considered isolated, since the design process consists of an iteration of both. This is because, as equation (4) shows, the $\tilde{g}_i$ transfer functions, obtained with structural decomposition, incorporate the controller transfer functions in their definitions. Thus, every time a design is done for one of these transfer functions, it will be needed to recalculate the others. From the new $\tilde{g}_i$, a new controller could be designed, and then, definitely, the algorithm needs an alternation between the two phases.

4. EXAMPLES

One of advantages that presents this methodology of multivariable controller design is to be independent of the process model. This has been shown in numerous examples in (Vázquez *et al, 1*999), which can be repeated with TITO tool. Also, the model does not have reason to be a rational expression in the Laplace operator, but rather it can be a frequency response array. These characteristics allow the immediate extension of this method, using it accompanied by some decoupling strategies (one of the described in section 2 or another), so that in a first step the decoupling matrix is obtained and, in a second step, the controllers for the decoupler+process block designed. Some of the possibilities are shown in following examples.

Example 1: Following shows a system model with a RGA next to 1.7. Some decoupling strategy could be tested. It is a distillation column (Vinante and Luyben, 1972) that describes the existent dynamics between reflux and vapor flow and the temperatures of plates 4 and 17:

$$\begin{pmatrix} R \\ V \end{pmatrix} = \begin{pmatrix} \frac{2.2}{7s+1}e^{-s} & \frac{1.3}{7s+1}e^{-0.3s} \\ \frac{2.8}{9s+1}e^{-1.8s} & \frac{4.3}{9.2s+1}e^{-0.35s} \end{pmatrix} \begin{pmatrix} T_4 \\ T_{17} \end{pmatrix} \quad (6)$$

In order to analyze the effects of designing with decouplers, expressions (1) are used. In this case, dynamic and steady state decoupling coincide:

$$d_{11}=1, \quad d_{12}=-0.59, \quad d_{21}=-0.65 \quad \text{and} \quad d_{22}=1 \quad (7)$$

Design specifications are phase margin (PM) of 45º and gain margin (GM) of 4 for both loops, and looking for a PID with a limit of the relationship between the derivative and integral constant (α) equal to 0.01.

The design using iterative algorithm without decouplers gets following controllers:

$$k_1 = 0.88\left(1+\frac{1}{1.82s}+0.035s\right) \quad \text{and}$$
$$k_2 = 2.70\left(1+\frac{1}{1.79s}+0.073s\right) \quad (8)$$

Desing specifications are met with combined (PM and GM) tuning design in three iterations. Interaction impedes using other methods based on Gershgorin bands, as Ho's method.

Next, a complete decoupling is chosen. Design specifications are the same as before (PM=45º and GM=4 for both loops). The algorithm converges in same number of iterations as before, obtaining following controllers

$$k_1 = 0.964\left(1+\frac{1}{2.81}+0.057s\right) \quad \text{and}$$
$$k_2 = 0.967\left(1+\frac{1}{0.766s}+0.014s\right) \quad (9)$$

With this design, specifications are also met, but now, controlled system was decoupler + process block. Time responses are shown in figure 3. Response without decouplers is also superimposed.

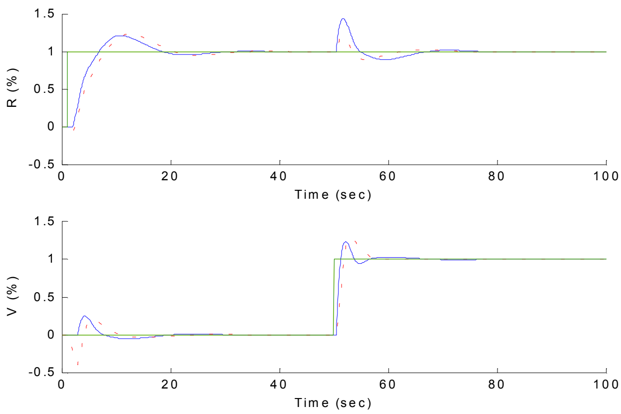

Fig. 3: Time response of example 1 with decoupler and without decouplers (continuous)

A slight modification can be appreciated, mainly in second loop, and an interaction decrease on the first one. Although modifications in time response are minimum, design is good to show methodology effectiveness when decouplers are incorporated.

Example 2: Table 1 shows the results of PID controller design for the system

$$\begin{pmatrix} X_D(s) \\ X_B(s) \end{pmatrix} = \begin{pmatrix} \dfrac{12.8}{16.7s+1}e^{-s} & \dfrac{-18.9}{21s+1}e^{-3s} \\ \dfrac{6.6}{10.9s+1}e^{-7s} & \dfrac{-19.4}{14.4s+1}e^{-3s} \end{pmatrix} \begin{pmatrix} F_R(s) \\ F_S(s) \end{pmatrix} \quad (10)$$

for different decoupling nets (dynamic, in steady state, partial and total) and for different controllers (PI, PID). All the designs have been carried out with specifications of PM=60º and GM=4 for both loop. It is a water-methanol distillation column (Wood and Berry, 1973) analyzed in numerous later works (Ho *et al*, 1996), (Toh and Devanathan, 1993), (Vázquez *et al*, 1999) (it appears in the practical whole of multivariable control references).

d_{12}	d_{21}	Kp_1	Ti_1	Td_1	Kp_2	Ti_2	Td_2
0	0	0.51	12.62	0	-0.027	2.46	0
18.9/12.8	0	0.48	8.05	0	-0.030	8.94	0
0	6.6/19.4	0.35	9.82	0	-0.031	2.39	0
18.9/12.8	6.6/19.4	0.34	6.11	0	-0.027	9.19	0
$\frac{18.9(16.7s+1)}{12.8(21s+1)}$	0	0.49	8.72	0	-0.038	9.90	0
0	$\frac{6.6(14.4s+1)}{19.4(10.9s+1)}$	0.33	7.60	0	-0.033	2.925	0
$\frac{18.9(16.7s+1)}{12.8(21s+1)}$	$\frac{6.6(14.4s+1)}{19.4(10.9s+1)}$	0.32	5.67	0	-0.031	8.08	0
$\frac{18.9(16.7s+1)}{12.8(21s+1)}$	$\frac{6.6(14.4s+1)}{19.4(10.9s+1)}$	0.34	4.91	0.34	-0.046	9.626	0.759

Table 1: Comparative of different decouplers and designs of example 2

Figure 4 shows superimposed time responses of the system without decouplers and with total dynamic decouplers, with a PI and a PID (the two last designs of table 1). The PID design has been carried out limiting α to 0.25.

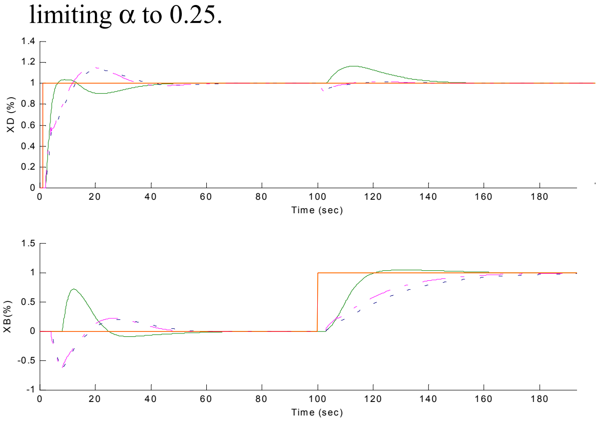

Fig. 4: Time response with decouplers (PI, and PID) and without decouplers (continuous).

Note that interaction effects decrease not very significantly. This same reduction could be obtained tuning the second loop in a less aggressive way (for example PM=80º), which would reduce its effect on first loop.

Example 3: This third example studies a process proposed by Niederlinski (1971) and analyzed with a set of decouplers proposed by Shiu and Hwang (1998). The process function transfer matrix is**:**

$$\begin{pmatrix} y_1(s) \\ y_2(s) \end{pmatrix} = \begin{pmatrix} \dfrac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \dfrac{-1}{(0.1s+1)(0.2s+1)^2} \\ \dfrac{1}{(0.1s+1)(0.2s+1)^2} & \dfrac{2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \end{pmatrix} \begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix} \quad (11)$$

Shiu proposes the following decouplers, obtained after identifying the $\tilde{g}_i$ functions applying a relay method:

$$d_{12} = \frac{2(0.587s+1)(0.0676s+1)}{(0.379s+1)(0.2x0.0676s+1)} \quad \text{and}$$

$$d_{21} = -\frac{0.417(1.06s+1)(0.197s+1)}{(0.48s+1)(0.2x0.197s+1)} \quad (13)$$

In order to compare time responses with and without decouplers, the tuning algorithm has been used with the same design specifications (PM = 60º for both loops).

Without decouplers, controllers are

$$k_1 = 1.11\left(1+\frac{1}{0.67s}\right) \quad \text{and}$$

$$k_2 = 0.27\left(1+\frac{1}{0.88s}\right) \quad (13)$$

obtained in six iterations of the algorithm. With decouplers, the controller equations are

$$k_1 = 0.91\left(1+\frac{1}{0.23s}\right) \quad \text{and}$$

$$k_2 = 0.26\left(1+\frac{1}{0.37s}\right) \quad (14)$$

obtained in only three iterations, because, in general, the algorithm converges quicker the more dominant the system is.

Figure 5 shows time response, with and without decouplers. Note that interaction decreases. If this fact is quantified by means of some measure as the IAE test, it can be proven that in the response without decouplers, IAE is 3.42, while when decouplers are applied it is 0.98.

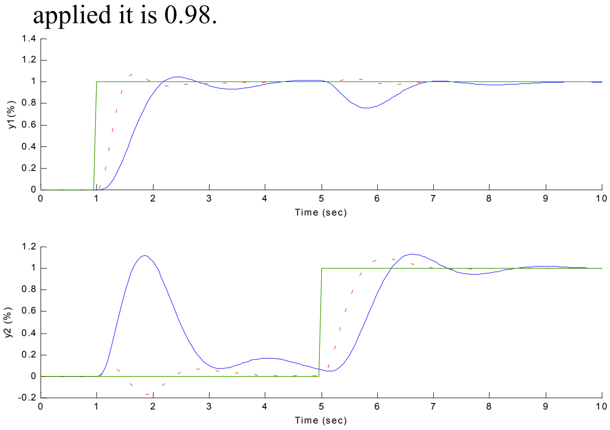

Fig. 5: Time response of the system of example 3, with and without decouplers (continuous).

Gerhgorin bands and SDNA of the process with and without decouplers can also be compared in figures 6 and 7. Note that dynamic interaction effects are reduced significantly.

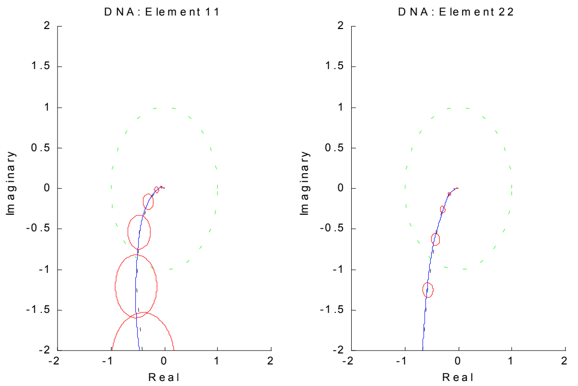

Fig. 6: SDNA and Gershgorin bands of system of example 3 with decouplers

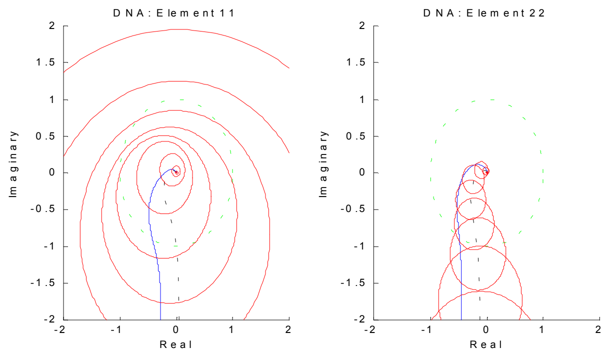

Fig. 7: SDNA and Gershgorin bands of system of example 3 without decouplers.

CONCLUSIONS

This work has been developed to design multivariable controllers including a decoupling net. It takes advantage of methodology proposed in (Vázquez *et al,* 1999), and it does not need a transfer function matrix with the system model but a representation with frequency response. Independently of how decouplers had been obtained, they can be included between process and controllers. Then, a decentralized PID controller is designed, using some of the different possibilities of proposed algorithm: design with only phase margin specifications for both loops, only gain margin, or a combination of PM and GM specifications.

All designs have been obtained with TITO tool, a set of MATLAB m-functions that can be obtained in web page http:// www.uco.es/~in2vasef.

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