# A NEW METHOD OF SELF-TUNING DIGITAL PID CONTROLLERS

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Abstract. This paper presents a new method of self-tuning digital PID controllers. This method uses a technique of pattern recognition; it supposes a first-order model for the process and uses the quarter decay ratio on the error signal as tuning criterion for the PID parameters. The theoretical foundation of the method is the study of the root-locus for the closed-loop system. The PID parameters are set, according to the characteristics of the process model, so that the root-locus have a prescribed form. As the simulations shows, the method is suitable to a wide class of typical processes, p.e. those which transfer functions can be characterized by a steady-state gain, one time constant and a dead time.

Keywords. PID process control; direct digital control; pattern recognition; self-tuning controllers; adaptive control.

## INTRODUCTION

Nowadays there are two different ways to implement self-tuning digital PID controllers. One of them follows the work of Astrom and Wittenmark (1973) which uses techniques of identification and stochastic control. The other is based in the classical work of Ziegler and Nichols (1942) which is still used widely in the industry. This method uses a technique of pattern recognition and has the advantage of being very simple and robust; recently, Foxboro (Kraus and Myron, 1984) has introduced a family of industrial regulators based on this technique.

The self-tuning method that is proposed in this work is included within the last class: the PID parameters are fitted by watching the error signal when perturbations, set point o load changes, on the system happen. The tuning criterion is to get quarter decay ratio on the error signal. As in the open-loop method of Ziegler and Nichols (1942) it is supposed a first-order model for the process. Also the new method is a closed-loop method that does not require the process to undergo sustained oscillations, as the method of Chidambara (1970).

The theoretical foundation of the method is the study of the root-locus for the closed-loop system. The extension of the classical tuning methods to the discrete case do not give good results, as the root-locus shows. The method contribute with a new form of tuning PID digital parameters when quarter decay ratio on the error signal is required. As the simulations show, the method is suitable to a wide class of typical processes.

The paper is organized as follows: in the first place, the assumptions that the method makes are briefly described. Next the self-tuning procedure is given. Results from simulations with typical processes using classical tuning methods and the new method are discussed next.

## PROPOSED METHOD FOR TUNING PID REGULATORS

The method is based on the following four assumptions.

(a) The process model. It is supposed a first-order model for the process that is based on the process step response shown in Fig. 1, which is typical of many real processes. Hence, the process model transfer function can be written as

$$G_m(s) = K \frac{\exp(-T_d s)}{T_p s + 1} \quad (1)$$

where a first estimate of the steady-state gain K, the time constant $T_p$, and the dead-time $T_d$ may be obtained from the knowledge of the process or from some off-line experiment as is illustrated in Fig. 1.

(b) Digital PID control algorithm. It is used the digital PID control algorithm proposed by Auslander and others (1978) which can be written in its position form as

$$u(k) = -K_P y(k) + K_I \sum_{i=0}^{k} (r(i)-y(i)) - K_D(y(k)-y(k-1)) \quad (2)$$

The coefficients $K_P$, $K_I$ and $K_D$ are the proportional integral and derivative gains, $K_I=K_P T/T_I$, $K_D=K_P T_D/T$ where $T_I$ and $T_D$ represent the integral and derivative constants and T the sampling period.

The main characteristic of this algorithm is that in order to avoid sudden changes in the control variable when the set point is changed, only the I-action acts on the signal error while the P-action and D-action act on the process output.

(c) The sampling period. It is chosen according to the initial process model. It is advised that the time constant of the process model have to be comprised between ten and twenty times the sampling period and that the dead time have to be minor or equal than five sampling periods

$$\frac{T_p}{20} \leq T \leq \frac{T_p}{10} \quad (3)$$

$$T_d \leq 5\ T \quad (4)$$

when it is not possible to verify both conditions, the condition (3) is chosen.

If it is supposed that the dead time of the process model is always an integer multiple of the sampling period, $T_d = D\ T$, the corresponding discrete transfer function of the process model becomes

$$G_m(z) = K \frac{(1-\exp(-T/T_p))}{z^D(z-\exp(-T/T_p))} \quad (5)$$

When the digital PID control algorithm done by (2) is used with the condition $T_D = T_I/4$, the system characteristic equation can be arranged as

$$1 + K_R \frac{(z-c)^2}{z^{D+1}(z-p)(z-1)} = 0 \quad (6)$$

where $c = 1/(1+2T/T_I)$, $p = \exp(-T/T_p)$ and $K_R = K_p\ K\ (1-p)\ (1+T/T_I+T_I/4T)$. Hence if the ratio $T_I/T$ is set according to the characteristics of the process model, it will see later, the influence of the proportional gain $K_p$ in the closed-loop system response may be studied using the root-locus method.

(d) Tuning criterion. The PID parameters are tuning to achieve quarter decay ratio in the signal error when set point or load changes happen. This tuning criterion has been used for years, Ziegler and Nichols (1942), Chidambara (1970), and still is very used in process control.

## Self-Tuning Procedure

The process is being controlled by a PID algorithm with the characteristics supposed before, the PID parameters have been determined according to a first estimate of the process model. The self-tuning mode begins only when the signal error exceeds a nominal noise threshold and follows the next steps.

Step 1. Pattern recognition of signal error. The PID parameters remain fixed, but an algorithm begins to watch the signal error with the purpose of measure the decay ratio (b/a) and the time between peaks (N) as a multiple of sampling periods. A certain degree of validity is assigned to these measurements when are compared with the typical response of a second-order system, see Fig. 2.

Step 2. Estimation of the dead time. It is supposed that:
- The form of the signal error is due to a dominant pair of complex roots of the closed-loop system.
- The time constant of the process model is always the same.
- Any change in the process may be interpreted as change on the gain and/or the dead time of the process model.

Then it is verified that:
- The complex roots, having the form $z = r\ \exp(\pm jx)$, can be determined by the measurements, b/a and N, which were made in the step 1.
- The dead time of the process model may be estimated using the next expression

$$(D+1)x + \arctan\frac{r\sin(x)}{r\cos(x)-p} + \arctan\frac{r\sin(x)}{r\cos(x)-1} - 2\arctan\frac{r\sin(x)}{r\cos(x)-c} = \pi \quad (7)$$

where $x = \frac{2\pi}{N}$, $r = (\frac{b}{a})^{1/N}$ and all is known except D. This expression is the result of to impose the phase condition to the system characteristic equation (6).

Step 3. Tuning of PID parameters. The ratio $T_D = T_I/4$, which has been used by others authors (Ziegler and Nichols, 1942; Chidambara, 1970), is always used in this paper. In this way the tuning of PID parameters is reduced to calculate two parameters, the integral constant and the proportional gain.

With this condition and the previous hypotheses, the value of $T_I/T$ determine the form of the root-locus for the closed-loop system. If $D \neq 0$, three forms for the root-locus are possible.

Form 1; when the zero z=c is to the left but far of the pole z=p. See Fig. 3a.

Form 2; when the zero z=c is to the left but near of the pole z=p. See Fig. 3b.

Form 3; when the zero z=c is between the poles z=p and z=1. See Fig. 3c.

Integral constant; the ratio $T_I/T$ is set, according to the new characteristics of the process model in order to get two objetives.
- The root-locus closed-loop system must have the form 1. Because this is the only form that guarantees the existence of dominant complex roots for different values of the proportional gain and therefore it is the only form that guarantees the pure oscillating shape of the signal error.
- Within the form 1, the quarter decay ratio response have to be as fast as possible.

A theoretical study using the root-locus method allows to find and tabulate the value of $T_I/T$ that, for each pair of $T_p/T$ and $T_d/T$ values, gets that the two before conditions are verified. The values obtained allows to affirm that the ratio $T_I/T$ must be function of the time constant and the dead time of the process model in disagreement with the tuning rules done by other authors, Ziegler and Nichols (1942), Takahashi and co-workers (1970), in which $T_I/T$ is only function of the dead time.

Proportional gain; the new proportional gain is determined as a relative present proportional gain change with the purpose to get quarter decay ratio in the error signal for future set point o load changes.

This relative change is function of the present integral constant value, the oscillatory error signal form observed in the step 1 and of the process model chacracteristics estimated in the step 2. The relative change in $K_p$ may be obtained as follows: the present proportional gain $K_p$ verify the next equation

$$K_p\ K\ (1-p)\ (1+T/T_I+T_I/4T) =$$

$$= \frac{r^{D+1} \left( (r\cos(x) - p)^2 + r^2 \sin^2(x) \right)^{1/2} \left( (r\cos(x) - 1)^2 + r^2 \sin^2(x) \right)^{1/2}}{(r\cos(x) - c)^2 + r^2 \sin^2(x)} \quad (8)$$

this equation is the result of to impose the magnitude condition to the system characteristic equation (6) with the same suppositions that in the step 2. The new proportional gain $K_{Pn}$ verify the expression

$$K_{Pn} K = K_{P1/4} \quad (9)$$

where $K_{P1/4}$ is the necessary proportional gain to get quarter decay ratio, if the process model gain was the unit. This value is conditioned by the integral constant determined before and have been tabulated for each pair of time constant and dead time values using the root-locus method.

Therefore the relative change in $K_P$ may be obtained as quotient of (9) and (8).

Step 4. PID parameters update. The present PID parameters are replaced by the new parameters determined in the previous step. Precaution is taken so that sudden changes on the control signal do not happen.

## SIMULATIONS RESULTS

In order to evaluate the self-tuning digital PID controller proposed, an extensive number of simulations have been performed. The conclusions of these simulations is that the method works properly and may be applied to processes having widely different transfer functions.

Two representative examples, reported in previous papers by other authors, are presented showing the advantages of the scheme proposed.

Example 1. The transfer function

$$G(s) = \frac{24}{(s+1)\ (s+2)\ (s+3)\ (s+4)} \quad (10)$$

represents a typical chemical process that was considered by Chidambara (1970).

According to the open-loop step response of this process, the transfer function may be aproximated by

$$G_m(s) = \frac{\exp(-\ 0.9\ s)}{1.30\ s + 1} \quad (11)$$

According to the assumption (c), T = 0.13 is an appropiate sampling period for this case.

The PID parameters determined by Chidambara (1970) for this process in order to get quarter decay ratio have been

$K_P = 3.33$, $T_I = 2$ and $T_D = 0.5$

The simulation results with this PID parameters are shown in Fig. 4a, where SP is the set point, MV the process output and ERR the error signal. The vertical scale is 0 - 100 for each variable and the horizontal scale represents the successive sampling instants.

According to the process model (11), we deduce

$K = 1$, $\frac{T_p}{T} = 10$, $\frac{T_d}{T} \cong 7$

The PID parameters initially chosen are

$K_P = 1.68$, $T_I/T = 7.33$, $T_D/T = 1.83$

Figure 4b shows the simulation results with this PID parameters.

Although the tuning criteria was the same in both cases, we realize a great disagreement in Fig. 4a between the system response to set point changes, sampling instants 100 and 200, and the system response to the load changes, sampling instants 300 and 400. With the tuning method proposed the system response is quite similar in both cases, Fig. 4b, maintaining a quarter decay ratio.

The explanation of this results may be found in the root-locus for the closed-loop system. The ratio $T_I/T$ used in Fig. 4a makes the root-locus have the form 2 and the ratio $T_I/T$ used in Fig. 4b makes the root-locus have the form 1.

Example 2. The transfer function

$$G(s) = \frac{1}{10\ s + 1}\ \frac{50}{30\ s + 1}\ \frac{0.016}{3\ s + 1} \quad (12)$$

considered by Smith and Corripio (1985) corresponds to a typical problem of temperature control. These authors determine the follow model for the process

$$G_m(s) = 0.8\ \frac{\exp(-\ 7.2\ s)}{54.3\ s + 1} \quad (13)$$

Using the assumption (c), the sampling period chosen is T = 2.715 . Then the process model is characterized by the following parameters

$K = 0.8$, $\frac{T_p}{T} = 20$, $\frac{T_d}{T} \cong 3$ and the PID parameters taken by the tuning method proposed are

$$K_P = 7.47,\quad T_I/T = 4.40,\quad T_D/T = 1.10 \quad (14)$$

Simulations results show that the system response to set point and to load changes have a decay ratio greater than 1/4.

Taking (14) as initial values of the PID parameters the algorithm of autotuning is executed when changes in the set point happen, see Fig. 5. TABLE 1 shows the initial and successives values of the PID parameters.

TABLE 1 Simulation results in Example 2

| $K_P$ | $T_I/T$ | $T_D/T$ |
|---|---|---|
| 7.47 | 4.40 | 1.10 |
| 6.94 | 5.78 | 1.45 |
| 6.93 | 5.78 | 1.45 |
| 6.92 | 5.78 | 1.45 |
| 6.91 | 5.78 | 1.45 |

These results may be justified because in the procedure of autotuning the estimate model of the process is given by

$K = 0.74$, $\frac{T_p}{T} = 20$, $\frac{T_d}{T} = 4$

## CONCLUSIONS

A simple method for self-tunig PID controllers has been presented and analysed in detail in this paper. The theoretical study of the method has been made using the root-locus technique. This has allowed to justify the limitations of the classical tuning to implement PID regulators based on the pattern recognition approach.

The new method which is based in the classical work of Ziegler and Nichols (1942) present the following significant advantages:

(a) The simulations reported indicate that the procedure of auto-tuning is robust and works quite well for a large class of processes.

(b) The algorithm does not require a great computational capacity. Therefore it may be incorporated in very simple industrial regulators.

(c) Reduce requirements for trained personnel.

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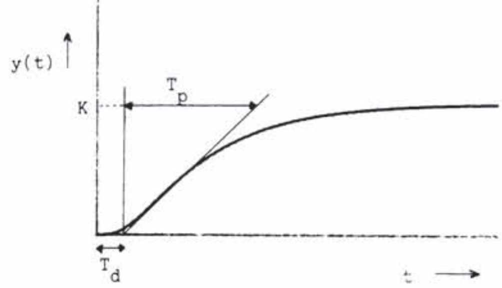


Fig. 1. Step response of typical processes.

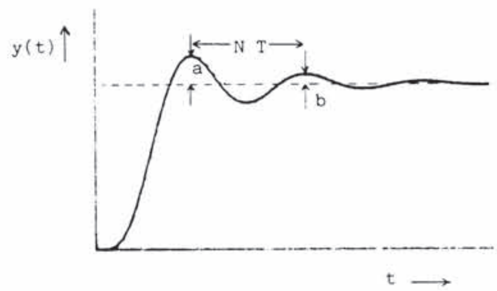


Fig. 2. Step response of closed-loop system.

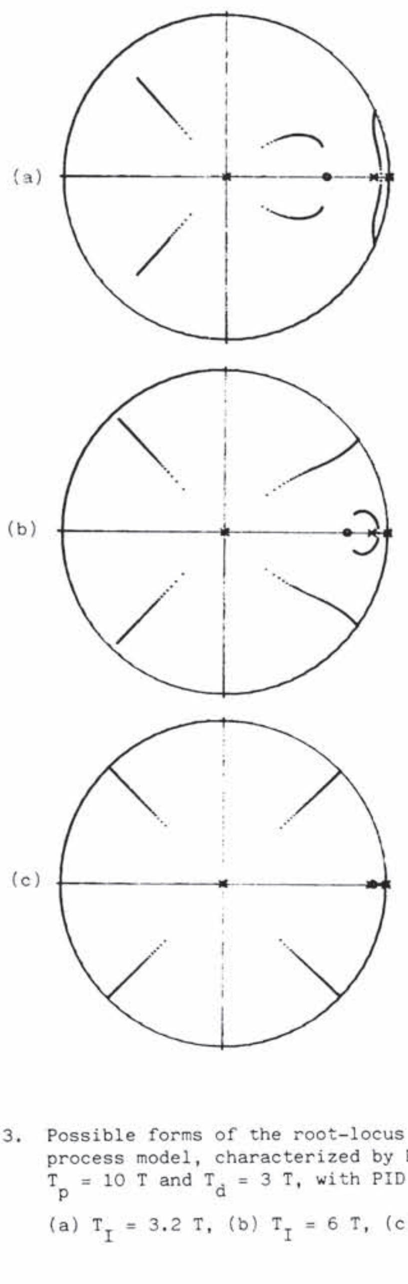


Fig. 3. Possible forms of the root-locus for the process model, characterized by $K = 1$, $T_p = 10\,T$ and $T_d = 3\,T$, with PID control.

(a) $T_I = 3.2\,T$, (b) $T_I = 6\,T$, (c) $T_I = 23\,T$

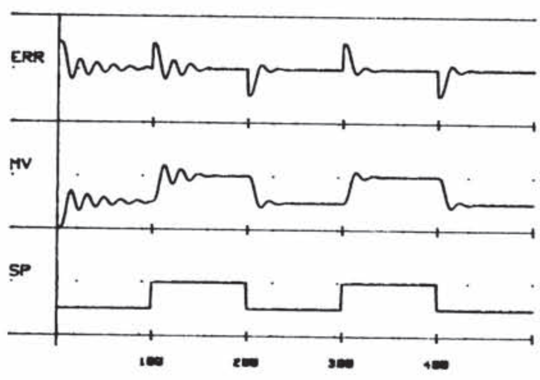


Fig. 5. Simulation of a self-tuning PID controller applied to a process with the transfer function G(s)=0.8/( (10s+1)(30s+1)(3s+1) )

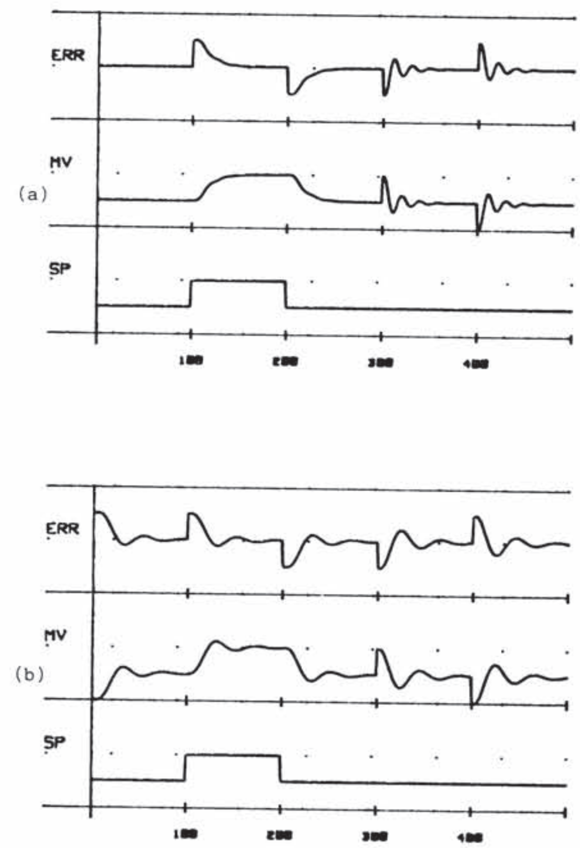


Fig. 4. Simulation of a PID controller applied to a process with the transfer function G(s)=24/( (s+1)(s+2)(s+3)(s+4) ).
(a) $K_P = 3.33$, $T_I = 2$, $T_D = 0.5$

(b) $K_P = 1.68$, $T_I = 0.96$, $T_D = 0.24$